

College Algebra Week 3
Chapter 4.1-4.4, 8.6, 8.7

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Where to from here?

- We have played with equations.
- We have played with inequalities
- We have played with the form of a linear equation $y = mx + b$.
- Secretly the m and b were very special. m is called the slope and b is called the y intercept.

Let's SEE what is happening

- One equation looks like the next until you make a picture
- “Why don't you take a picture, it will last longer!”

So let's build up our graphing skills

- First new definitions: *Ordered Pairs*
- They are the two numbers that make our special equation $y = mx + b$ work.
- They are the answers that go together.
- For $y = 2x - 1$ we can use $(2,3)$
(where we always say (x,y))
- $3 = 2(2) - 1 \rightarrow 3 = 3$ Remember doing this?

More than one answer

- If you change either x or y , then the other number changes. You can have **MANY** answers (and all those answers together make the graph we'll eventually get to).

Example 1

- Using $y = -3x + 4$
- Complete the ordered pair
- a) $(2, \quad)$ $y = -3 * 2 + 4 = -6 + 4 = -2$ $(2, -2)$
- b) $(\quad, -5)$ $-5 = -3x + 4$
 $-5 - 4 = -3x$
 $-9 = -3x$
 $3 = x$ $(2, 3)$

Example 1 continued

- $y = -3x + 4$
- c) $(0, \quad)$ Remember, this is just (x, y)
 $y = -3 * 0 + 4$
 $y = 4$ $(0, 4)$

The playing field

- The rectangular coordinate system
- (also called Cartesian)
- Or even the coordinate plane

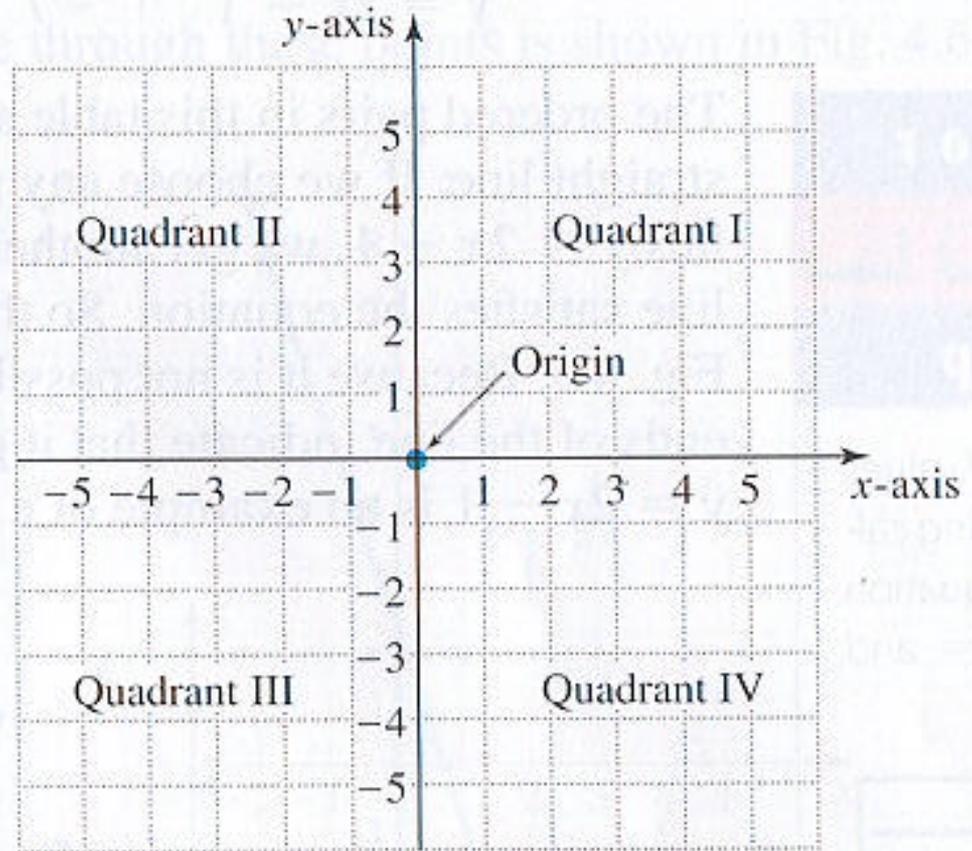


FIGURE 4.1

Parts is parts

- Important parts:
- The origin
- Quadrants
- X-axis
- Y-axis

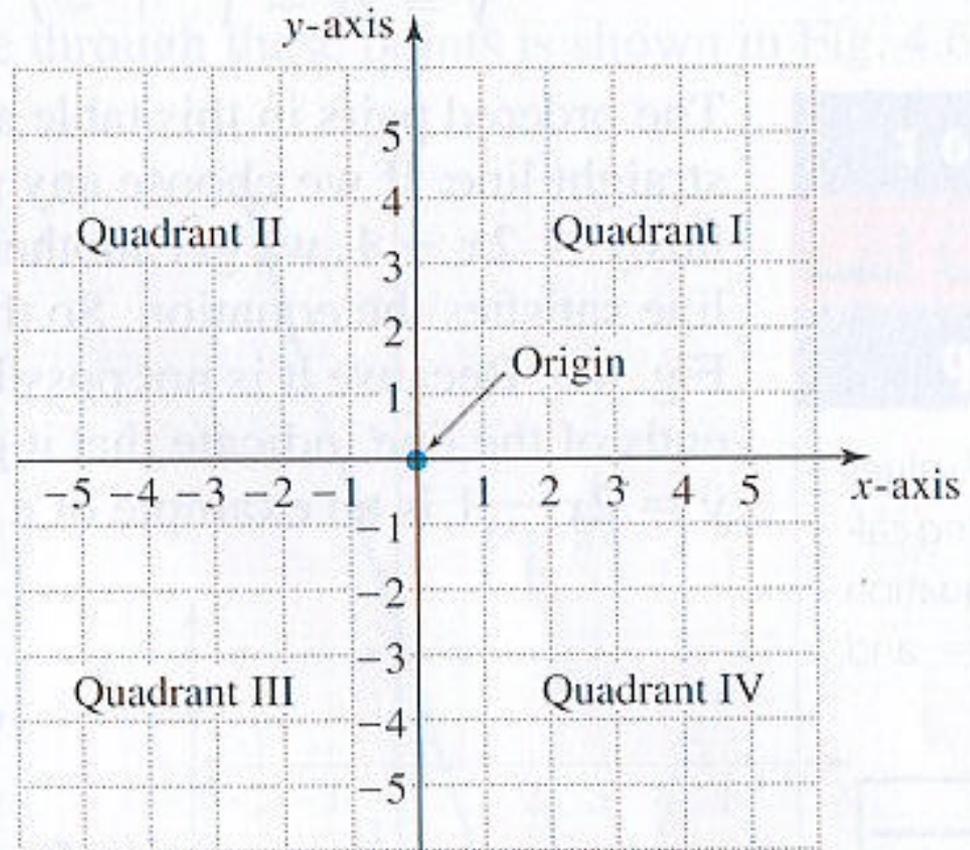


FIGURE 4.1

Quick helpful tips

- When you sketch one, make sure your hash marks for the numbers are equally spaced on each and both the X-axis and the Y-axis
- You don't have to plot the name of the points unless instructed to or if it is otherwise confusing

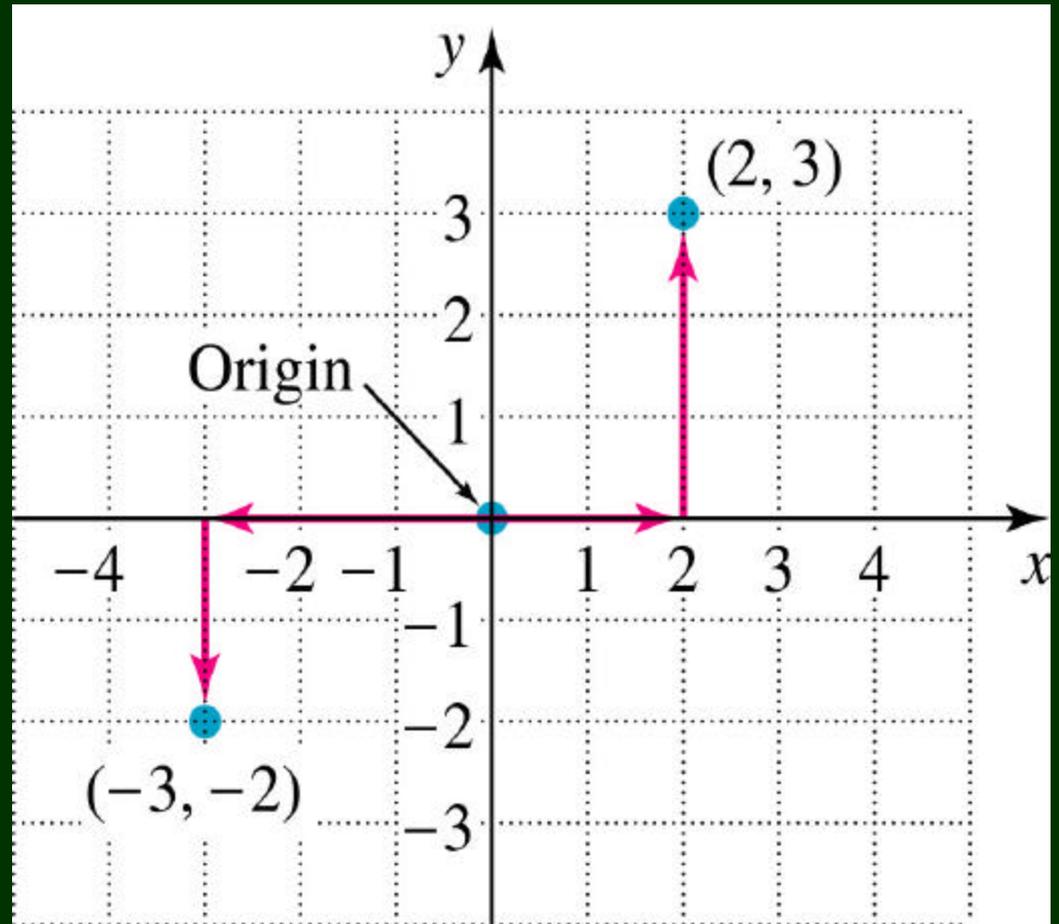
Plotting Points

- Just remember

(x, y)

Plot them!

- This is
 $(2,3)$ → top
- $(-3,-2)$
→ bottom



Example 2

- Plot the points
 $(2,5)$, $(-1,4)$,
 $(-3,-4)$, $(3,-2)$

EXAMPLE 2

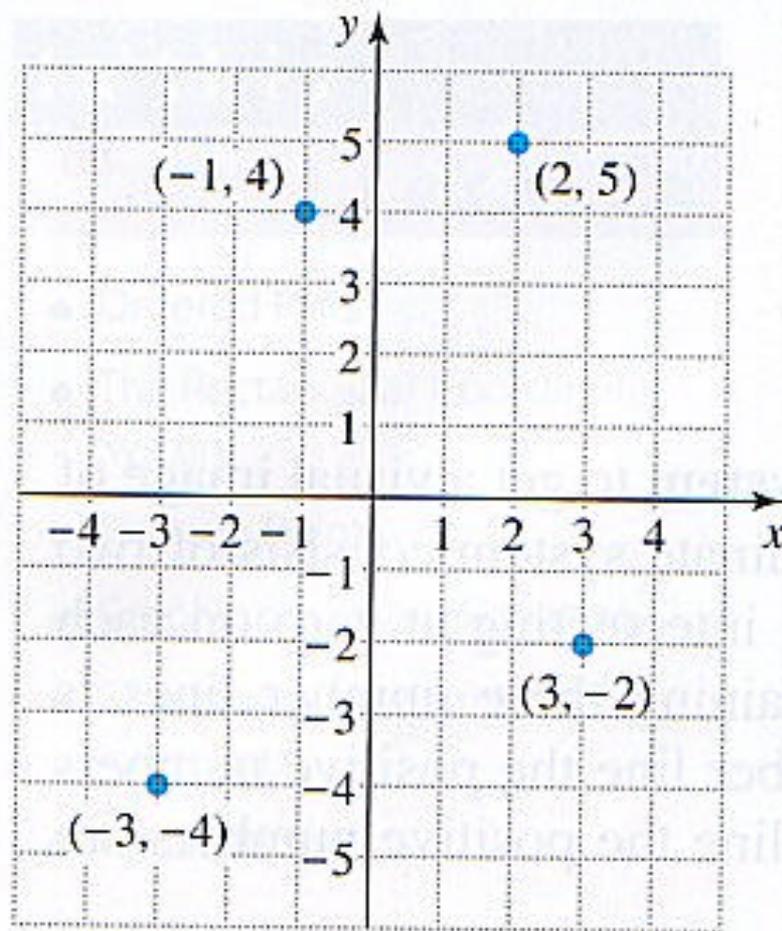


FIGURE 4.3

Graphing an equation (a linear equation for this class)

- If you have $y=2x-1$, you can try many different x 's and see what y 's you get, then plot all of them...

• If $x =$ -3 -2 -1 0 1 2 3

then

• $y=2x-1=$ -7 -5 -3 -1 1 3 5

This gives us
... plotted...

- Each x gave us a y from the equation $y=2x-1$

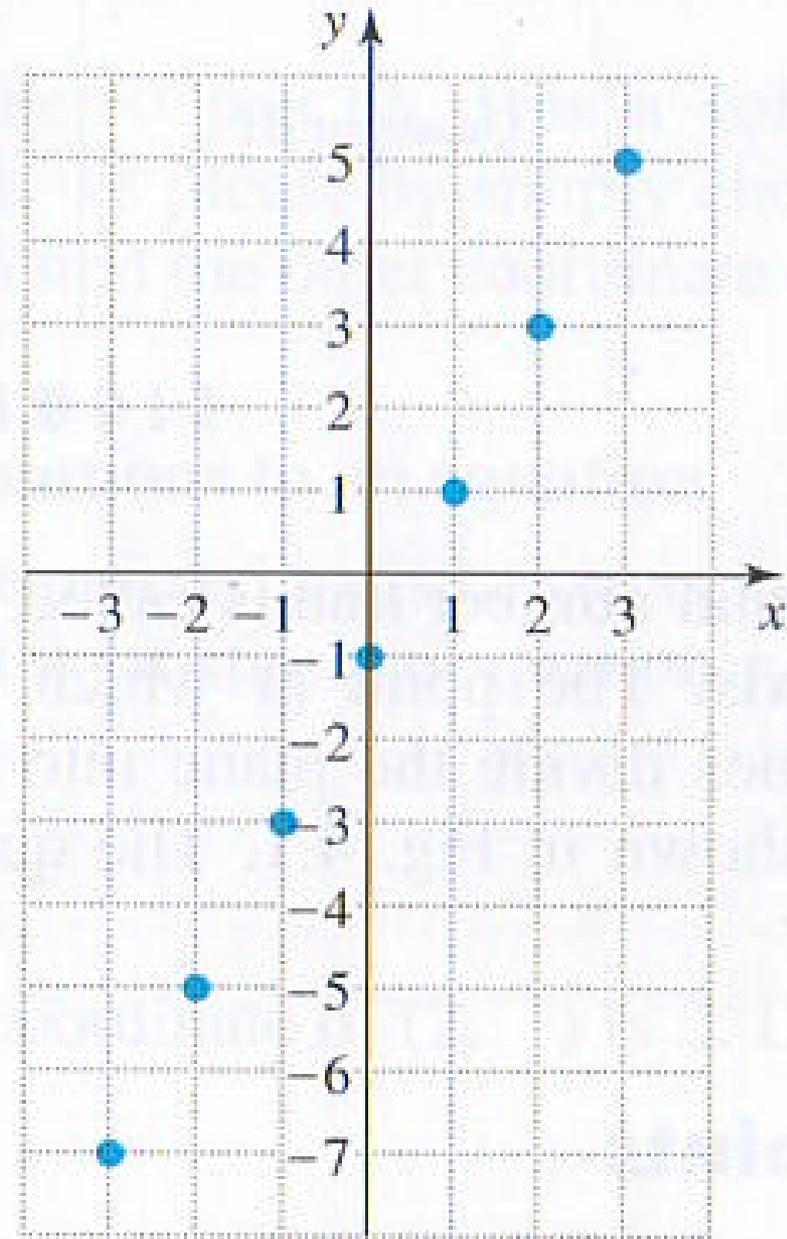


FIGURE 4.4

Then connect the dots

- And we write in the equation to tell later folk what is plotted

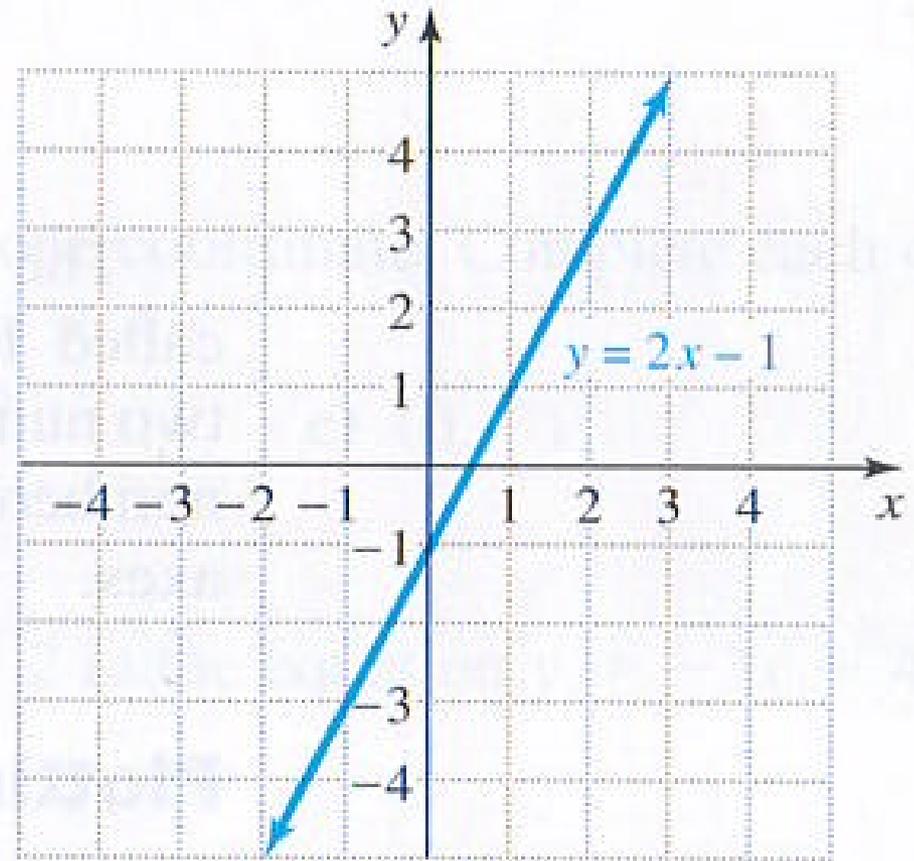


FIGURE 4.5

Definition

- The *Linear Equation in Two Variables*
- It is one written as $Ax + By = C$
- Where A, B, C are real numbers and both A and B cannot be $= 0$ at the same time.

Linear Equation in Two Variables

A **linear equation in two variables** is an equation that can be written in the form

$$Ax + By = C,$$

where $A, B,$ and C are real numbers, with A and B not both equal to zero.

A photo album of linear equations in two variables

- $x - y = 5$
- $y = 2x + 3$
- $2x - 5y - 9 = 0$
- $x = 8$ (B = 0 this time)

- We can solve all these to look like $y = mx + b$ and then graph them!

Example 3

- Graph $3x+y=2$
- Solve for y
- $y=-3x+2$
- Make our table

• If $x=$ -2 -1 0 1 2

Then

• $y=-3x+2=$ 8 5 2 -1 -4

Then graph them...

- Dot the dots
- Connect the dots

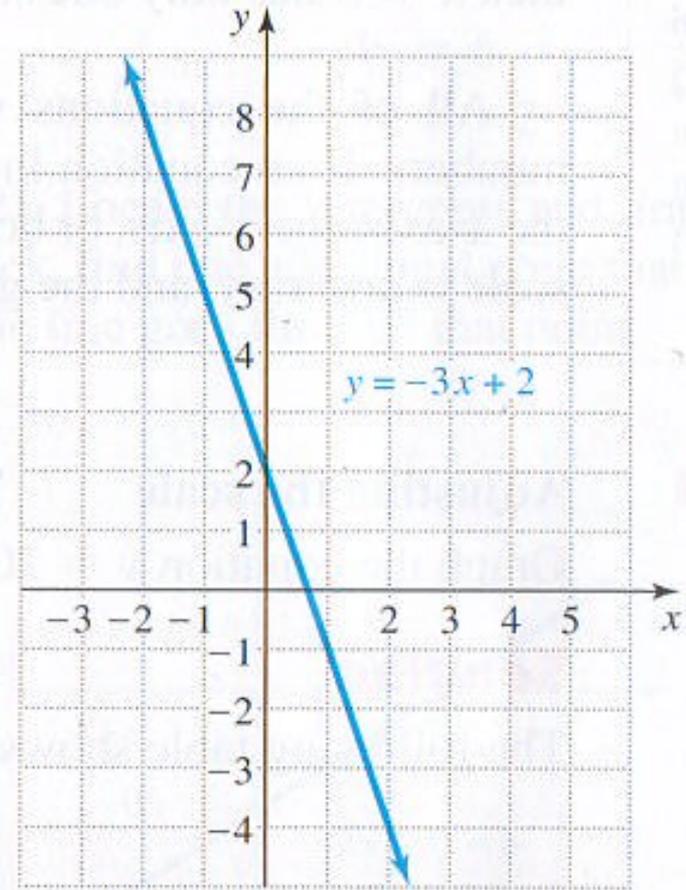


FIGURE 4.6

Example 4 The vertical line

- Take $x+0*y=3$
- Then $x=3$
- For $x=3$
y can equal
ALL NUMBERS

EXAMPLE 4

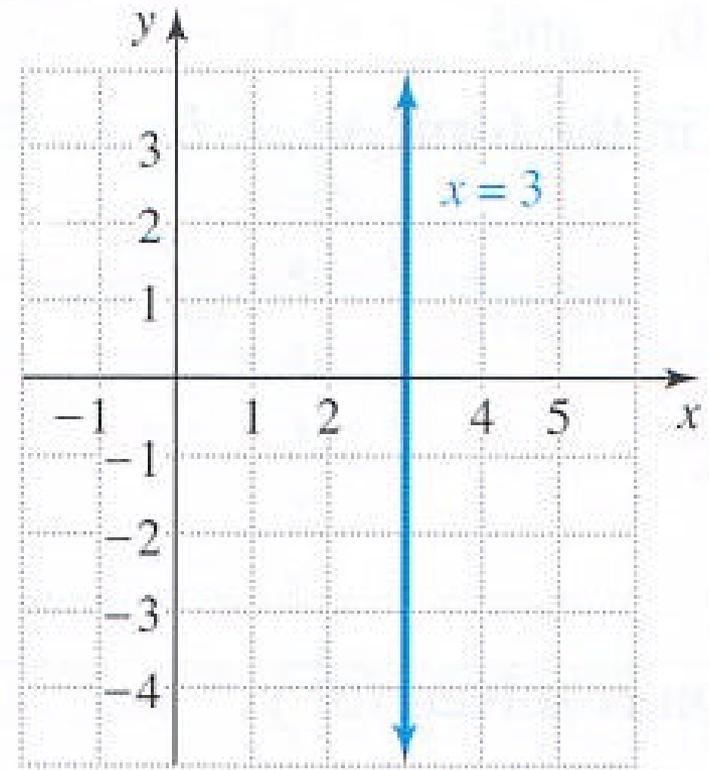


FIGURE 4.7

Example 5

- Adjusting the scale... what if -5 to 5 isn't enough on the graph labels?

- $y=20x+500$

- If $x =$ -20 -10 0 10 20

Then

- $y=20x+500$ 100 300 500 700 900

Plottin'

- The x and y axes CAN be different if it's needed
- Still even spacing!

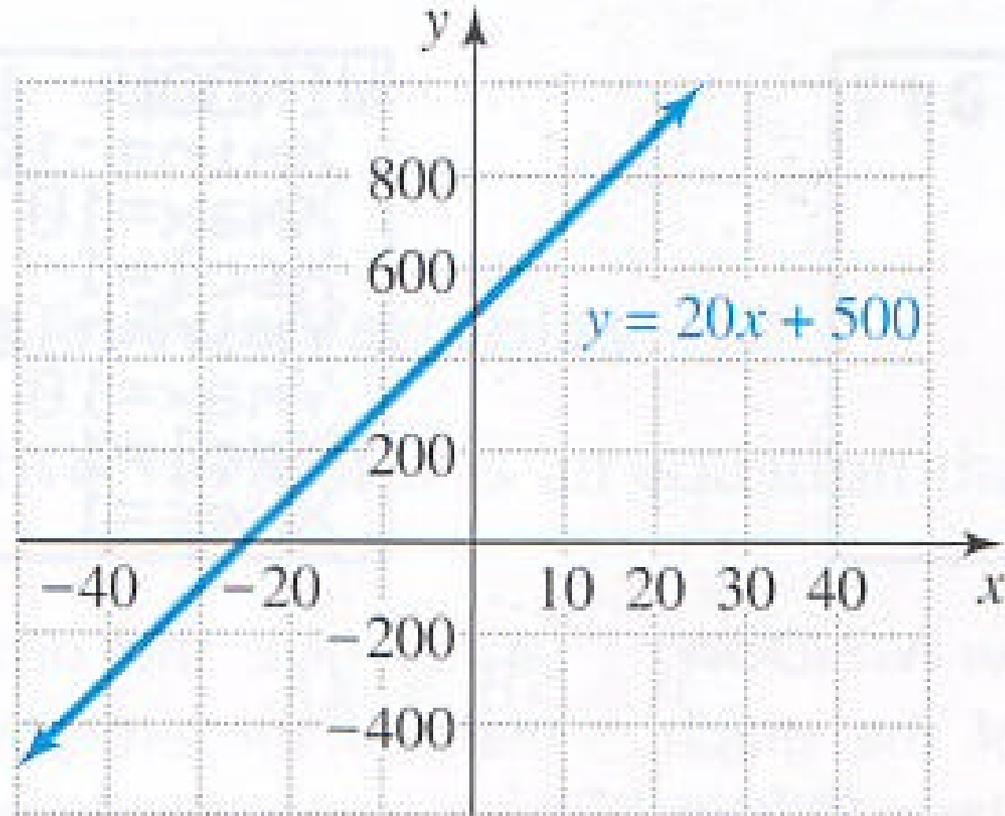


FIGURE 4.8

Now to the short cut

- Isn't making that table of plotted points a bit time consuming (read: boring?)
- How about a shortcut?!

Definitions

- Since 2 points make a line, let's just find two easy to find points, graph them and connect them.
- Job done
- We'll find the *x-intercept* (where the graph crosses X) and the *y-intercept* (where it crosses Y)

Example 6 – How it's done

- Graph $2x - 3y = 6$
- To find the x-intercept we MAKE $y=0$
(since that is what y equals on the x line)

- $2x - 3 \cdot 0 = 6$

$$2x = 6$$

$$x = 3$$

So our first point is $(3,0)$

Example 6 y-intercept

- Next plug $x=0$ into $2x-3y=6$ to find the y-intercept
(since $x=0$ on the y line everywhere)
- $2*0-3y=6$
 $-3y=6$
 $y=-2$ so our next point is $(0,-2)$

Plot the two points and draw!

- And put the equation in for others

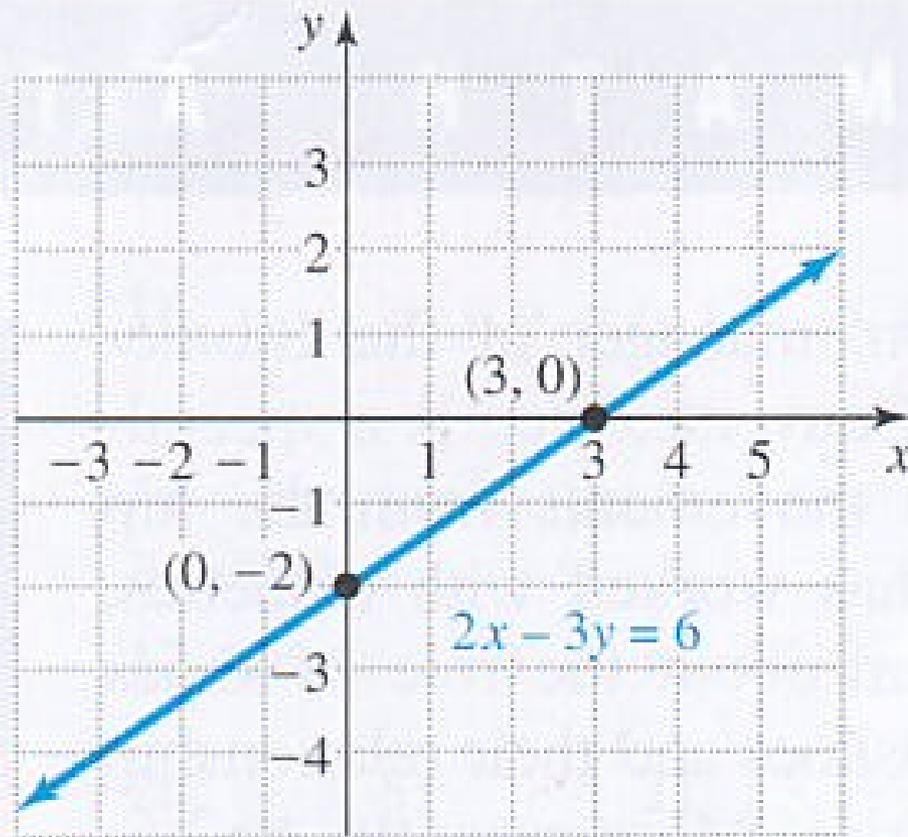


FIGURE 4.9

Application example 7

- We are given the equation that describes the demand for tickets for the Ice Gators hockey game
- $d=8000-100p$
- Where $d=$ the number of tickets sold, and p is the price in dollars

Example 7 continued

- a) How many tickets will be sold at \$20 per ticket?
- Plug 20 into the equation for p =price
- $d=2000-100*20=6000$
- So at \$20, we'd expect 6000 tickets

Ex 7 b

- Find the intercepts and interpret them
- Replace $d=0$ in $d=8000-100p$

$$0=8000-100p$$

$$100p=8000$$

$$d=80$$

- Replace $p=0$ in $d=8000-100p$

$$d=8000$$

We have (p,d) and $(0,8000)$ and $(80,0)$

If the price is free ($\$0$), we'll sell 8000 tickets (all)

If w up the price to $\$80$, we'll not sell ANY tickets

Ex 7c Graph it

- d) What happens to the demand as price increases?

As price increases, demand goes **DOWN**.

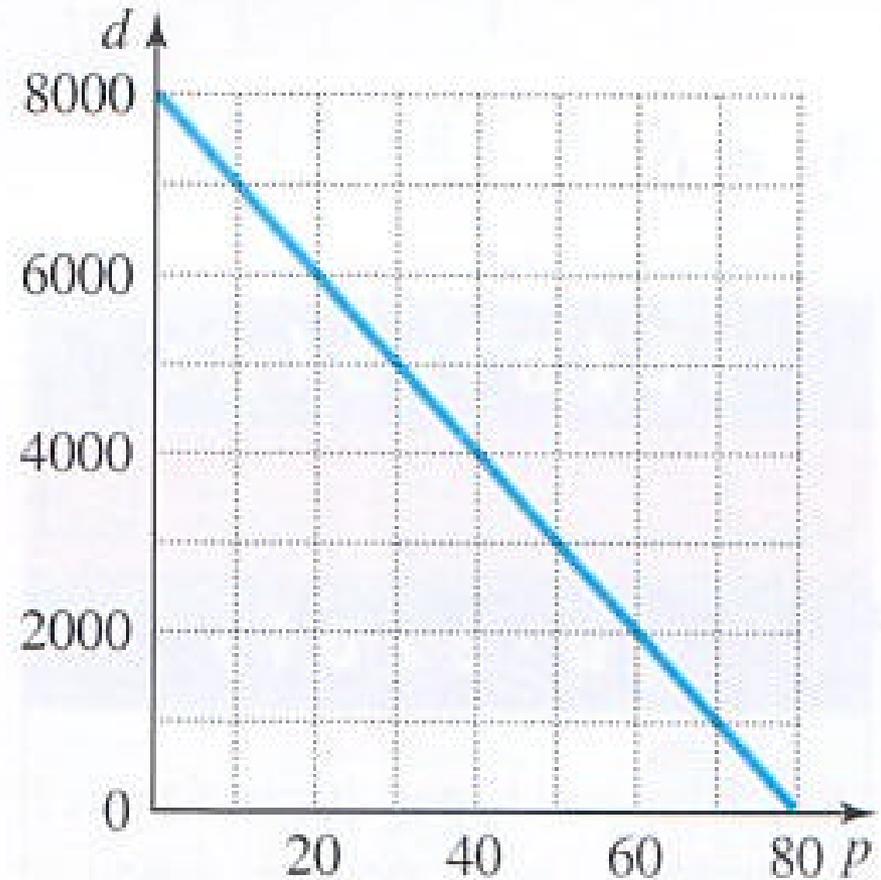


FIGURE 4.10

Try it! Section 4.1 Problems

- Try your hand at graphs...
- Definitions Q1-6
- Find the solution of the pairs Q7-16
- Plot simple points Q17-23
- Graph each equation plotting at least 5 points Q33-56
- Which Quadrant Game Q 57-68
- Graph each (can use x and y intercepts) Q69-82

Section 4.2 Looking closer at the 'm' → the SLOPE

- The slope and you
 - How fast are you climbing uphill?
 - For example, if you climb 6 feet for every 100 feet you drive forward, you are on a slope of $6/100$ or 6%.
 - It is how much you are climbing as you go out horizontally.
 - It is how much you change in y as you go out in x .

Sloping the Definition

- *SLOPE*

- Slope = the Change in the y-coordinate
the Change in the x-coordinate

Slope

$$\text{Slope} = \frac{\text{change in y-coordinate}}{\text{change in x-coordinate}}$$

Slope Examples

- Rise over Run! change in y over change in x
- Slope = $2/3$

Slope = $-2/-3 = 2/3$
same thing!

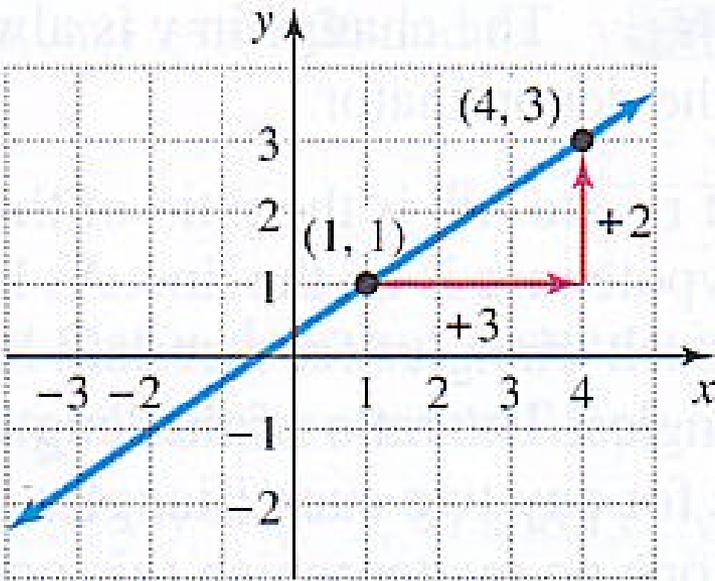


FIGURE 4.12

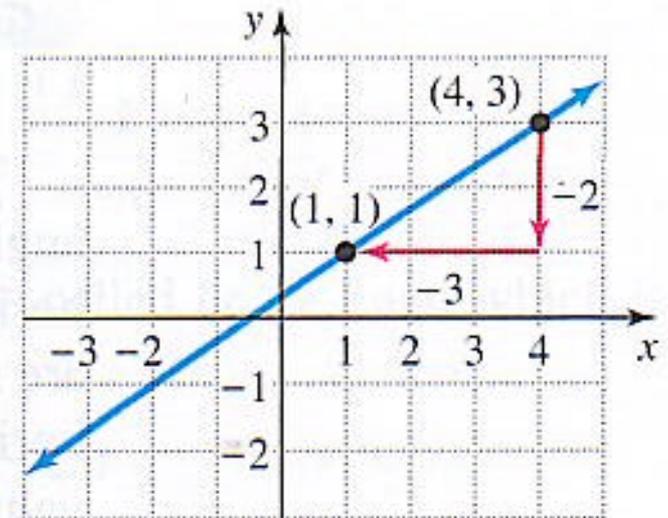
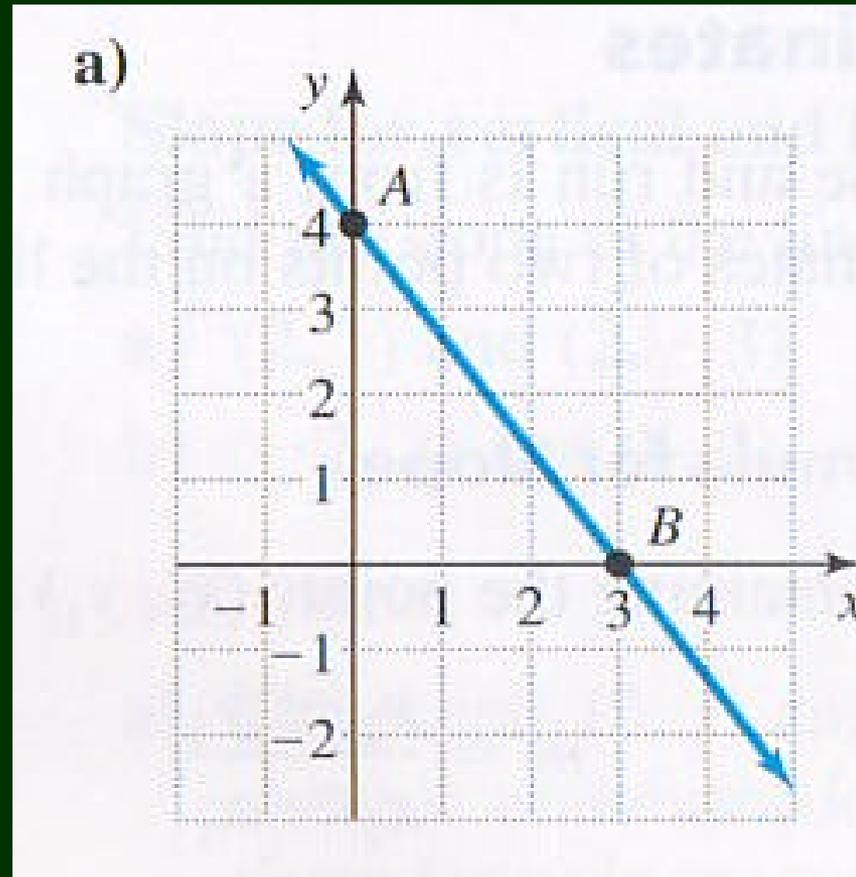


FIGURE 4.13

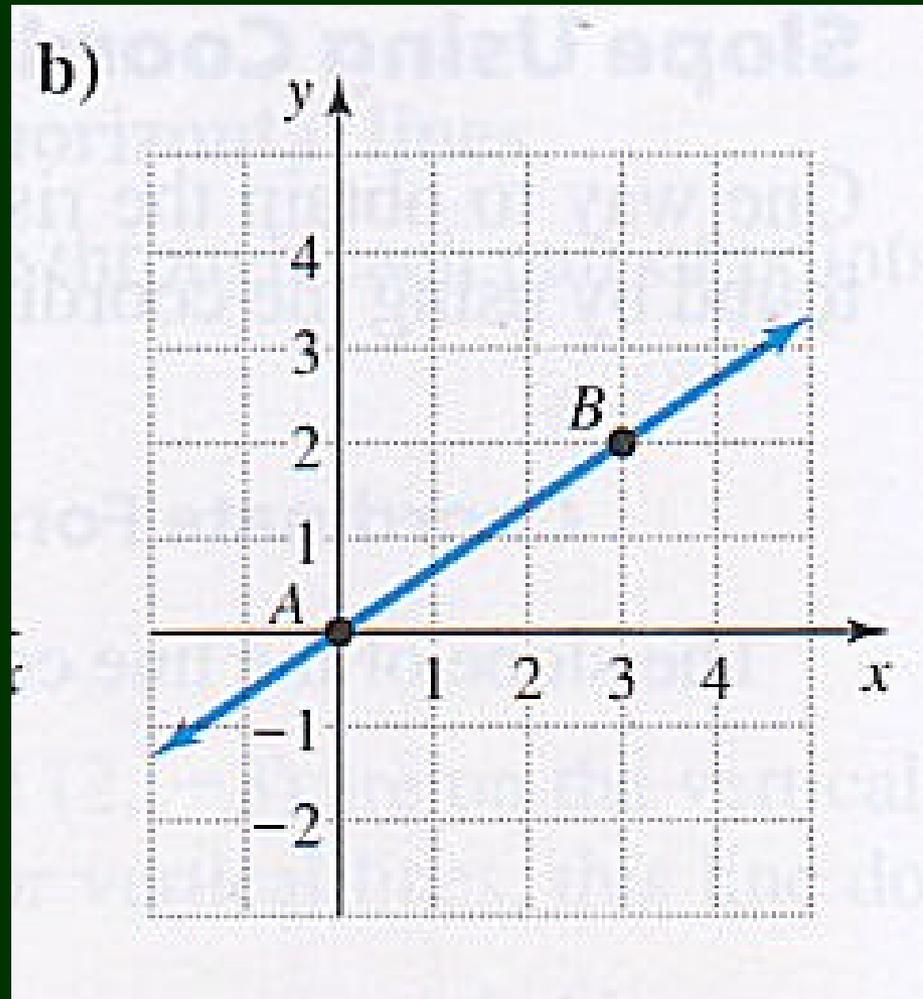
Example 1

- a) $m = -4/3$



Ex 1b

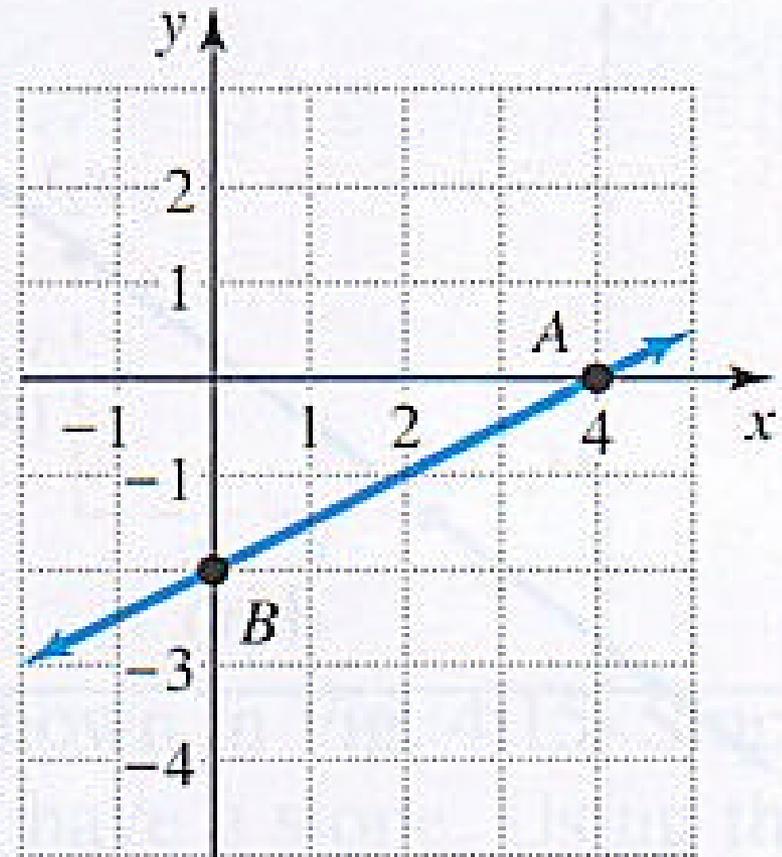
- b) $m=2/3$



Ex 1c

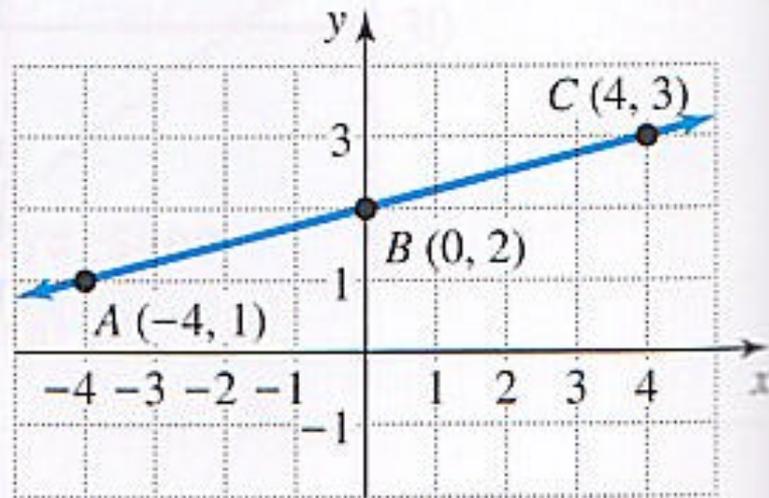
- $m = -2 / -4 = 1/2$

c)



Using ANY two points gives us the same slope! Ex 2

- We have points A, B, C. Let's find the slope between different pairs.
- A&B
 $m = \text{rise/run} = 1/4$
- A&C
 $m = \text{rise/run} = 2/8 = 1/4$
- B&C
 $m = \text{rise/run} = 1/4$



Now for a change in slope finding...

- Remember when we had answers to our $y=mx+b$ equation like (2,5) and (3,8)?
- We can find the slope from those too!

Definition

- The Coordinate Formula for Slope
(stick this in your memory!)

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)} \quad \text{This is just rise over run again.}$$

Provided $x_2 - x_1$ is not equal to zero.

Coordinate Formula for Slope

The slope of the line containing the points (x_1, y_1) and (x_2, y_2) is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1},$$

provided that $x_2 - x_1 \neq 0$.

Example 3a

- a) Find the slope of the following:
(0,5) and (6,3)
so (x_1, y_1) and (x_2, y_2)

$$m = (y_2 - y_1) / (x_2 - x_1) = (3 - 5) / (6 - 0) = -2/6 = -1/3$$

What if we reversed the points?

$$m = (y_2 - y_1) / (x_2 - x_1) = (5 - 3) / (0 - 6) = 2/-6 = -1/3$$

No difference. You can't go wrong!

Example 3b

- b) Find the slope of the following:
(-3,4) and (-5,-2)
so (x_1, y_1) and (x_2, y_2)

$$m = (y_2 - y_1) / (x_2 - x_1) = (-2 - 4) / (-5 - (-3)) = -6 / -2 = 3$$

Example 3c

- c) Find the slope of the following:
(-4,2) and the origin = (0,0)
so (x_1, y_1) and (x_2, y_2)

$$m = (y_2 - y_1) / (x_2 - x_1) = (0 - 2) / (0 - (-4)) = -2/4 = -1/2$$

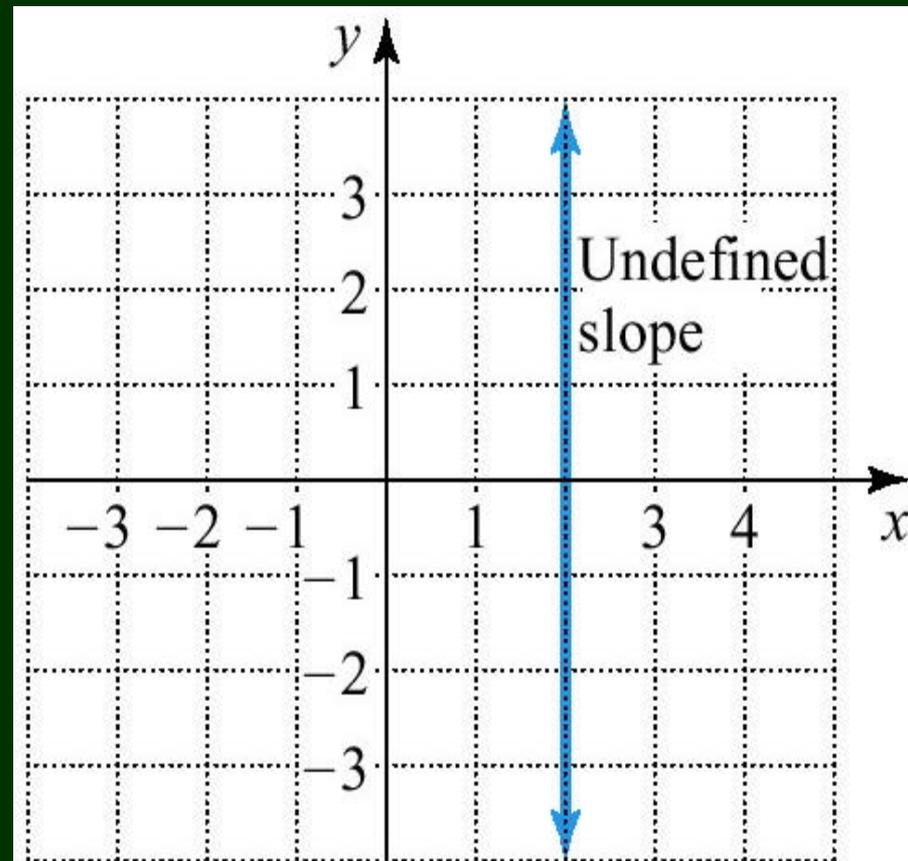
Caution... point 2 minus 1
don't mix the directions

$$m = \frac{y_2 - y_1}{x_1 - x_2} \quad \text{This will give the wrong sign.}$$

Example 4

What about vertical lines?

- 4a) (2,1) and (2,-3)
- $m = (-3-1)/(2-2) = -4/0$
- Explosion!
- Undefined slope
- Infinite slope

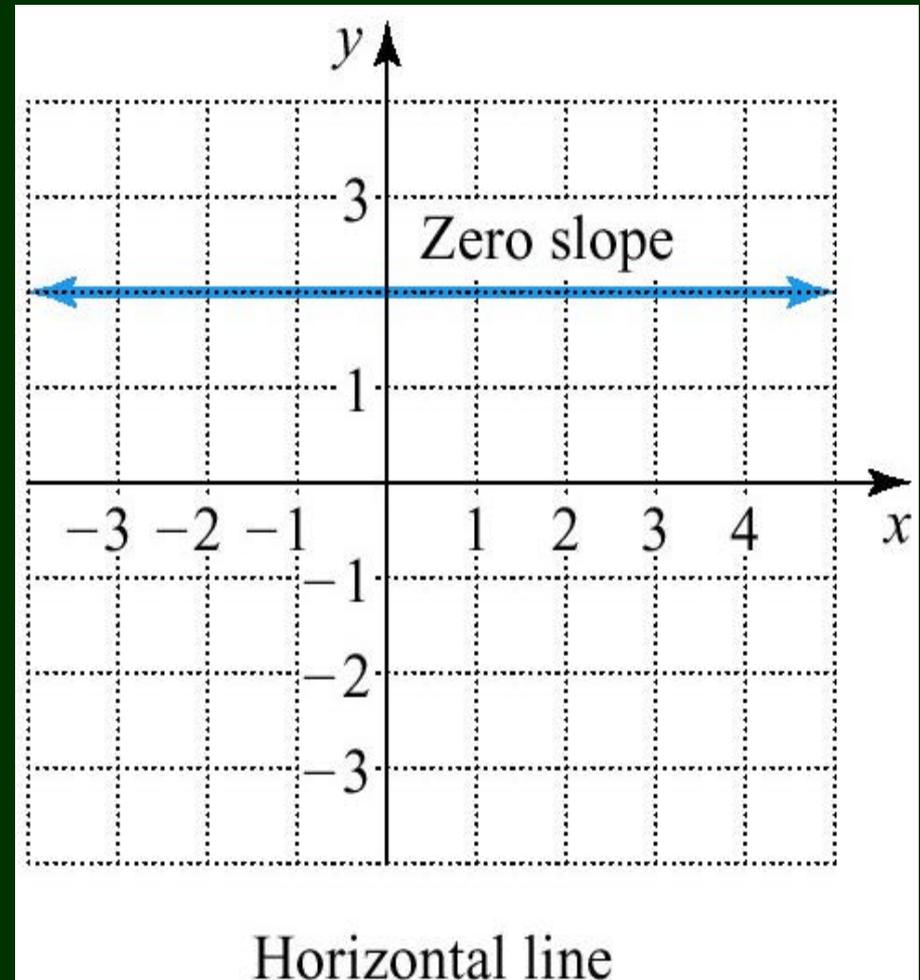


Vertical line

Ex 4b

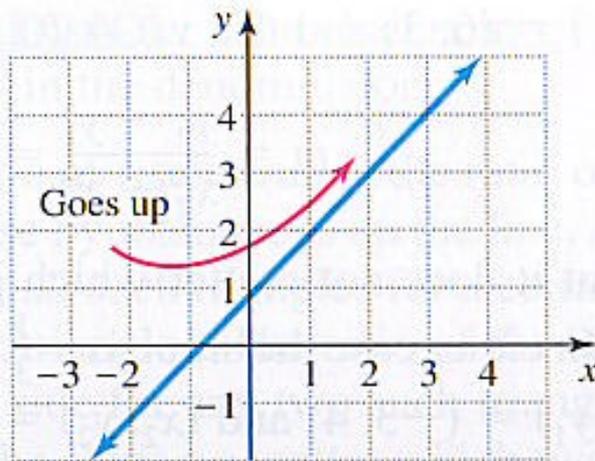
- $(-2,2)$ and $(4,2)$
- $m = \frac{2-2}{4-(-2)}$
 $m = \frac{0}{6} = 0$

Horizontal lines = zero slope

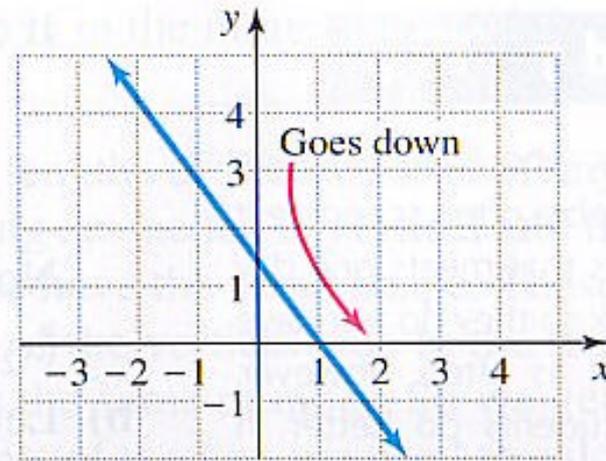


Checking sign at a glance

- If you see the line going UP as you read left to right, it is a **positive** slope.
- If you see the line going DOWN as you read left to right, it a **negative** sign.



Positive slope



Negative slope

FIGURE 4.17

Example 5 – Graphing a line if you have the slope and a point

- You are given a point (step 1- plot it!) then the slope (Step 2 :rise up then run out to the 2nd point, plot it!). Step 3- draw the line.
- Graph the line through $(2,1)$ with slope $3/4$

Graph the line through $(2,1)$ with slope $3/4$

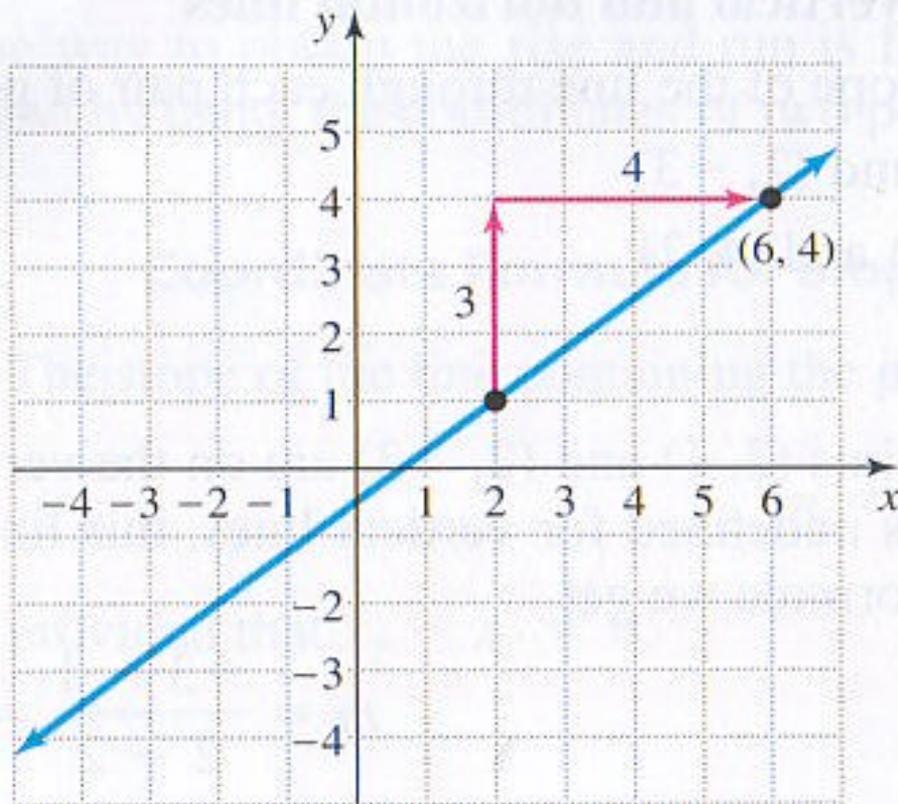


FIGURE 4.18

b) Graph the line through $(-2, 4)$ with slope -3

- The slope of -3 is $-3/1$ (same thing- right?)

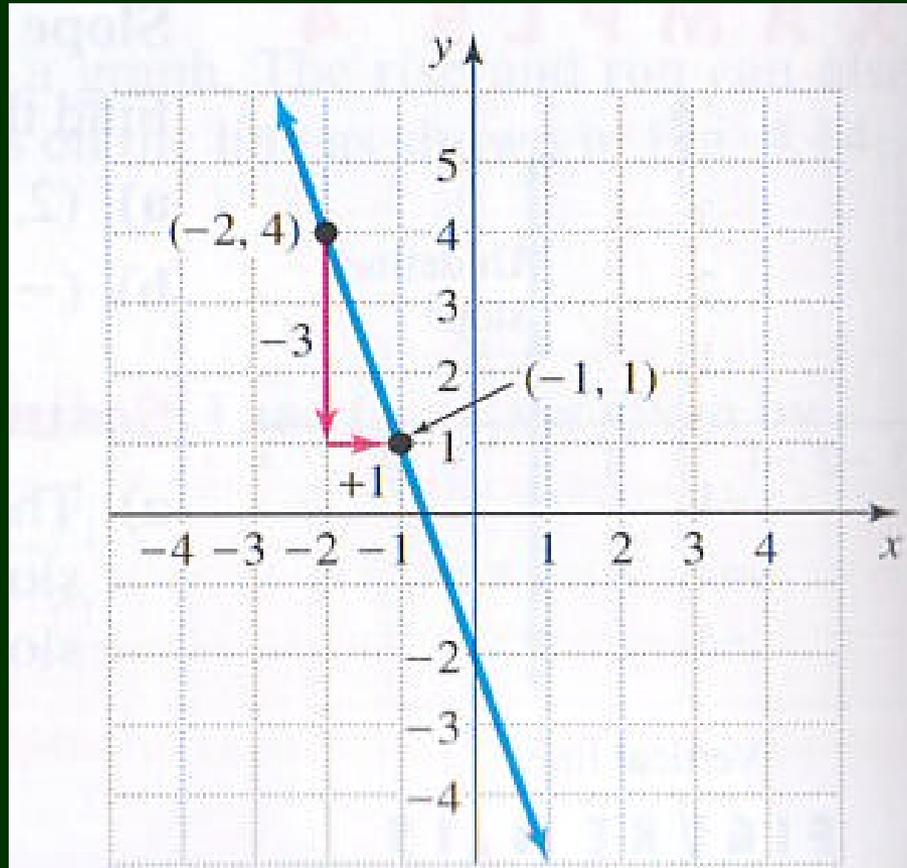


FIGURE 4.19

Parallel lines are lines with the same slope!

- Draw a line through $(-2,1)$ with slope $\frac{1}{2}$ AND another line through $(3,0)$ with slope $\frac{1}{2}$
- Same slope = parallel lines!
- Different anchor point!

Parallel Lines

Nonvertical lines are parallel if and only if they have equal slopes. Any two vertical lines are parallel to each other.

Example 6

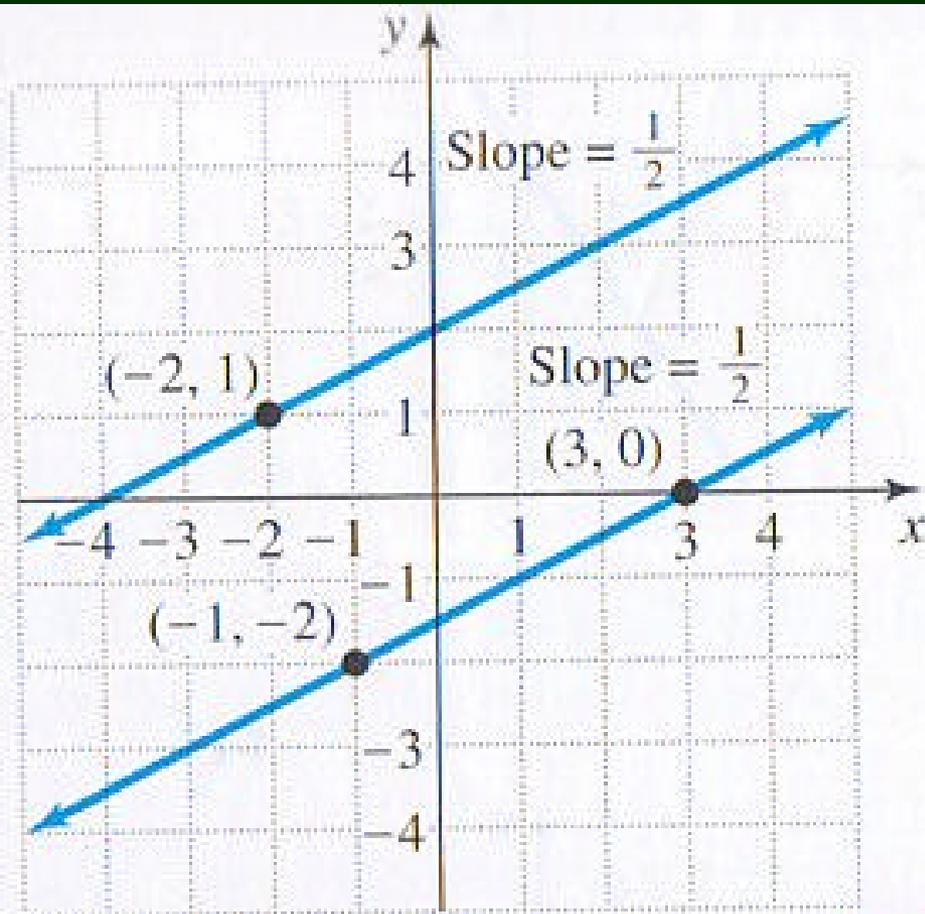


FIGURE 4.20



What about Perpendicular lines?

- Definition:

Perpendicular Lines

Two lines with slopes m_1 and m_2 are perpendicular ONLY if

$$m_1 = -1/m_2$$

Also, ANY vertical line is perpendicular to any horizontal line.

Restating it

Perpendicular Lines

Two lines with slopes m_1 and m_2 are perpendicular if and only if

$$m_1 = -\frac{1}{m_2}.$$

Any vertical line is perpendicular to any horizontal line.

Example 7

- Graphing Perpendicular lines...
- We'll draw them through point $(-1,2)$
- Slope 1 = $m_1 = -1/3$ Slope 2 = $m_2 = 3$

The graph on the next frame says it all...
(A picture is worth a 10^3 words.)

Perpendicular Graph

EXAMPLE 7

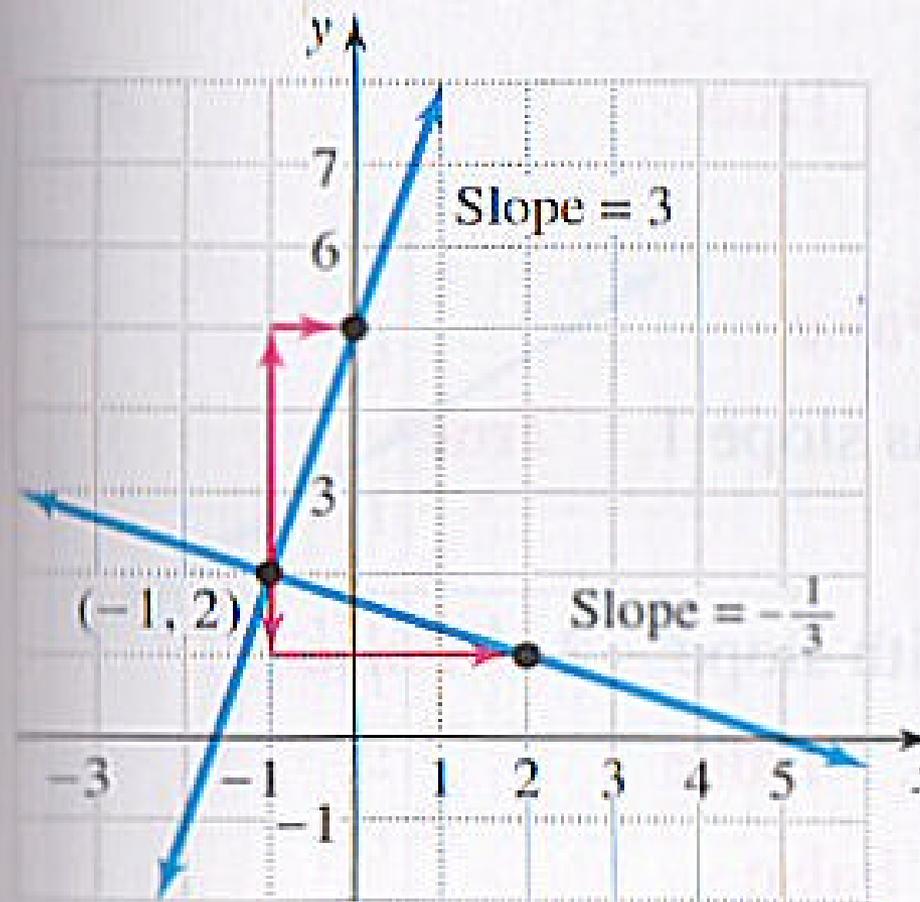
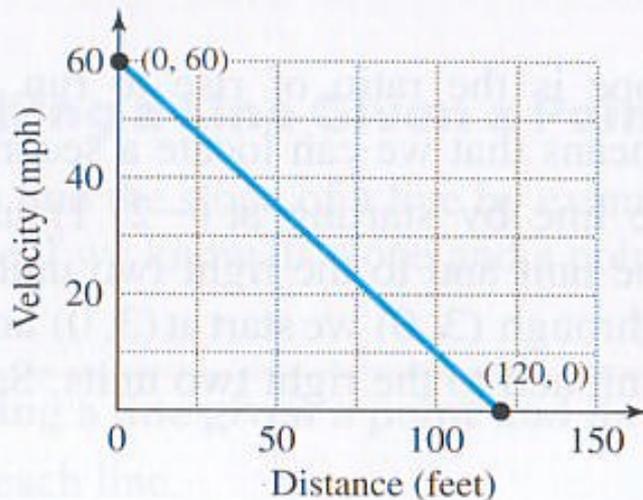


FIGURE 4.21

Using it in real life... Ex8

- If a car goes from 60mph to 0mph in 120ft... find the slope of the line.
- $m=(60-0)/(0-120)=-0.5$
- What is the velocity at 80feet?
- In 80 feet it drops $-.5(80)$ or -40 mph; $60-40=20$ mph



Slope Games in 4.2

- Yellow lines have homework problems...
- Definitions Q1-6
- Find slopes from plotted lines Q7-18
- Calculate the slope from points Q19-36
- Graph lines through a point with slope Q37-42
- Solve problem and graph Q43-52

Section 4.3 Putting $y=mx+b$ together with your slope knowledge

- What if we don't know a point, but know one point and the slope.
- We can play with the expression of slope to get $y=mx+b$
- $y=mx+b$ is soooo important, you should end up dreaming about this equation
- m = the slope, b = the y -intercept (where $x=0$)!

Getting to $y=mx+b$

- Given one point $(0,1)$ and the slope $2/3$, we call the other point (x,y) since we don't know what it is.
- Remember slope? $m=(y_2-y_1)/(x_2-x_1)$
- Plug in the numbers and x and y above
- We get $(y-1)/(x-0) = 2/3$
- Rewrite it $(y-1)/x = 2/3$ why write a 0?

continuing

- Our last line was: $(y-1)/x = 2/3$
- Now solve for y so we can get $y=mx+b$
- Multiply both sides by x
- $y-1 = 2/3 x$

Add 1 to both sides

- $y = 2/3 x + 1$ DONE, now we can graph it.
- We know that if $x=0$, $y=1$ so one point is $(0,1)$
[Hey! They told us that to begin with!] and $m=2/3$

Graphing it

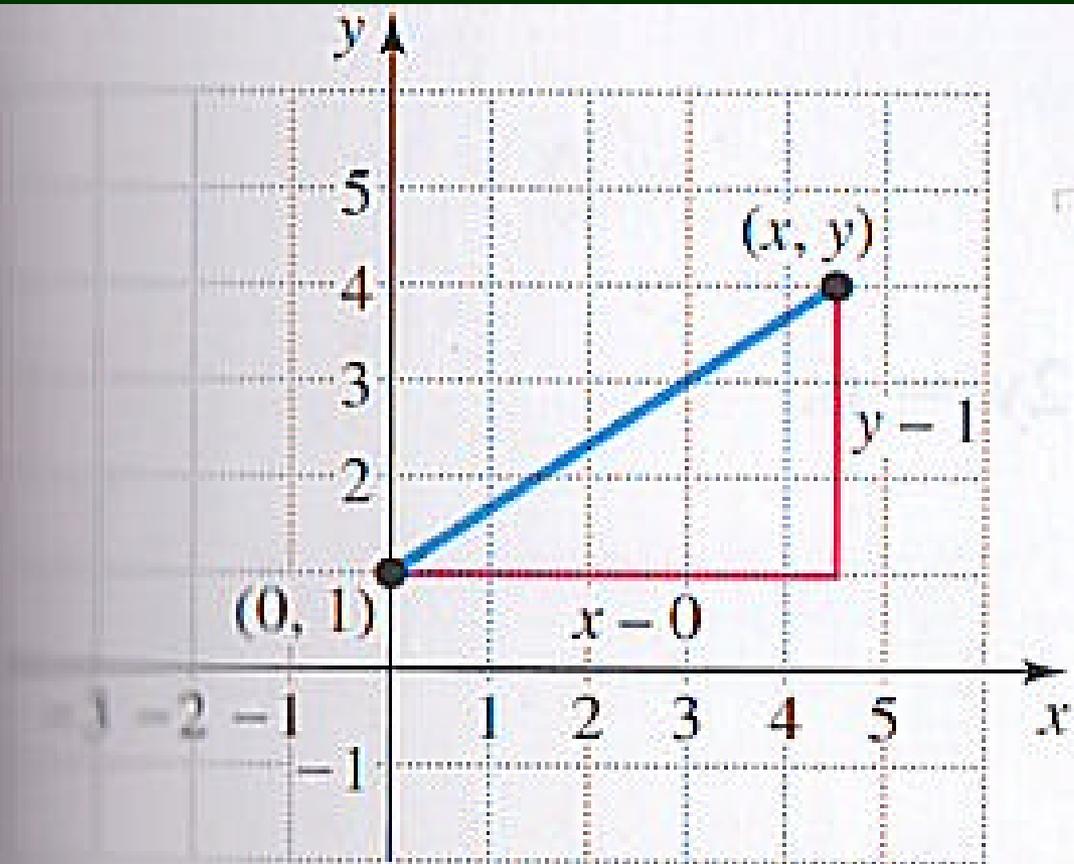


FIGURE 4.22

Definition Again!

- *Slope-intercept form*

$$y = mx + b$$

We know one point is always $(0, b)$ and the slope m gives us all we need to make ANY line!

Slope-Intercept Form

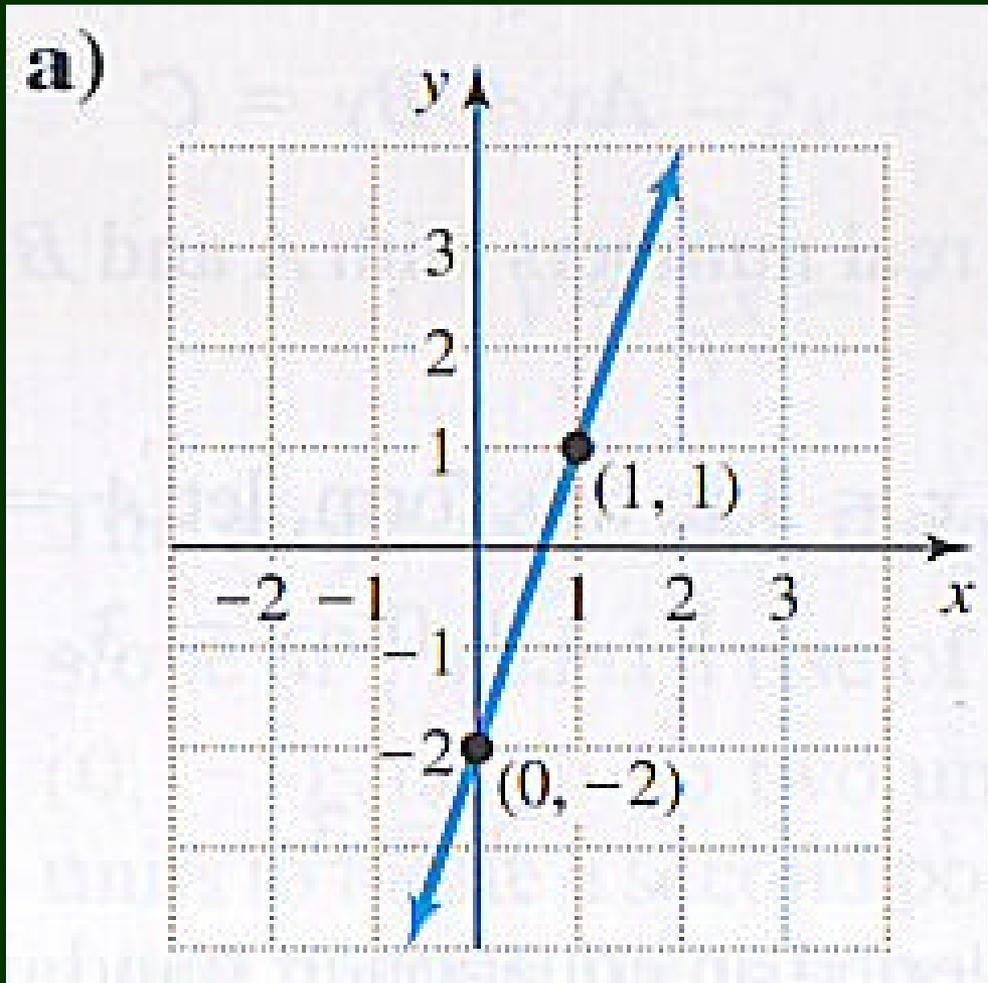
The equation of the line with y -intercept $(0, b)$ and slope m is

$$y = mx + b.$$

Example 1a

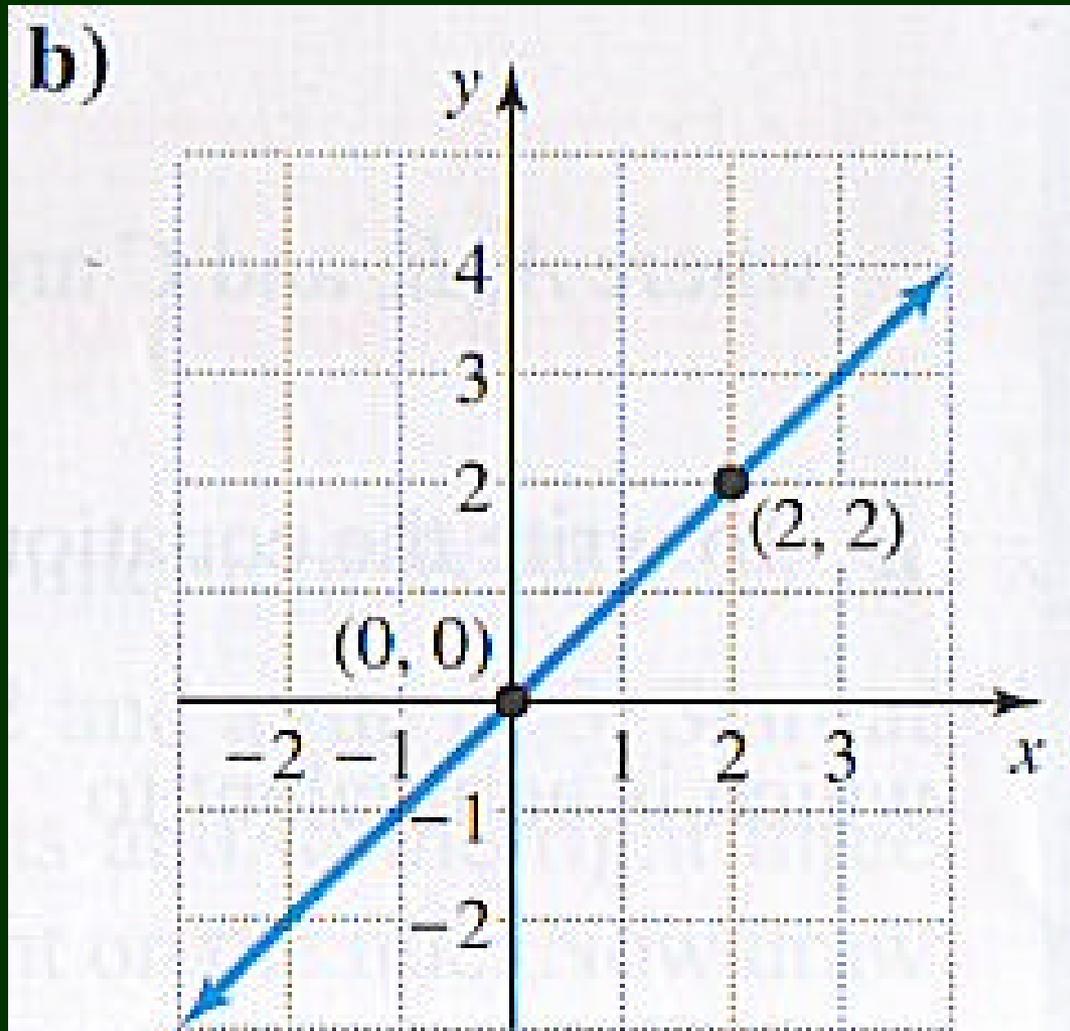
Using the slope-intercept form

- Write an equation from:
y-intercept=
(0,-2) so $b=-2$
- $m=3/1=3$
- $y=3x-2$



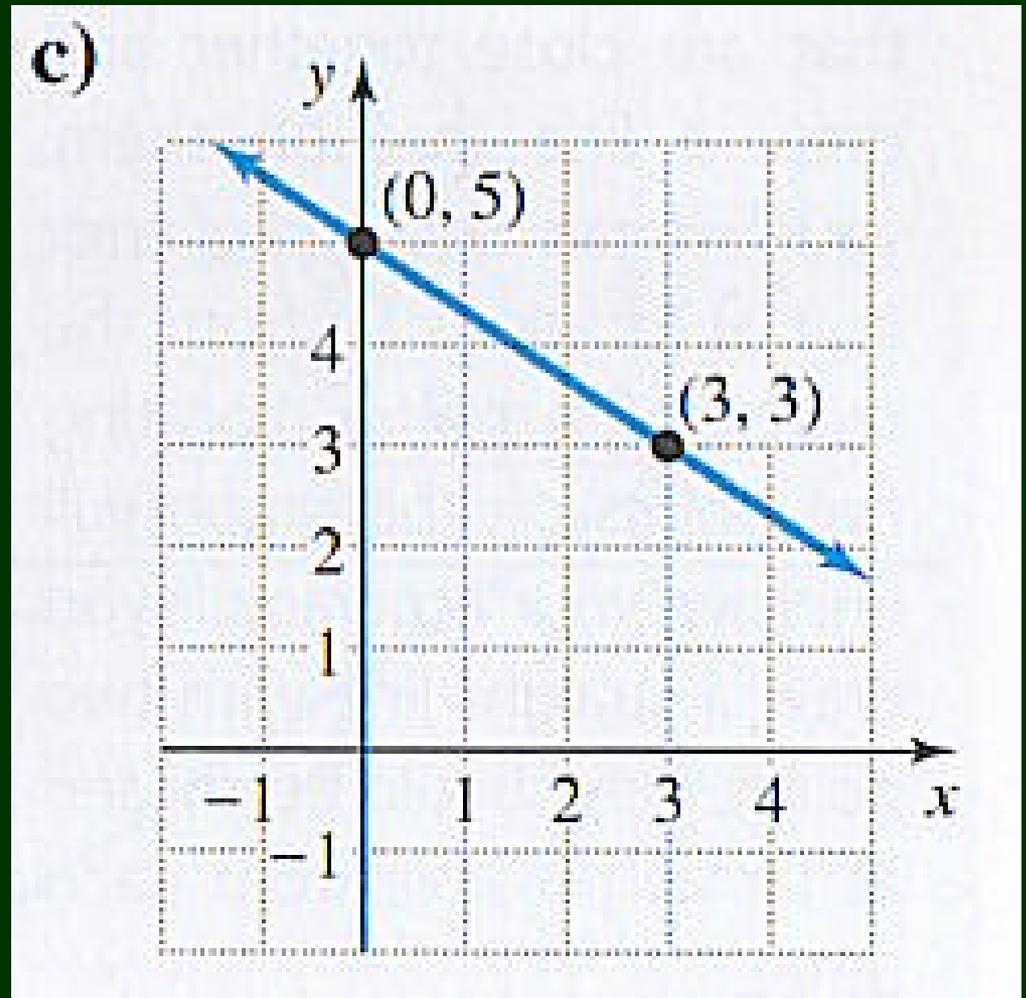
Example 1b

- Write an equation from:
y-intercept=
 $(0,0)$ so $b=0$
- $m=2/2=1$
- $y=1x-0$
 $y=x$



Example 1c

- Write an equation from:
y-intercept=
 $(0,5)$ so $b=5$
- $m=-2/3=-2/3$
- $y=-2/3x+5$
 $y=x$



Making it $y=mx+b$

- Example 2
- What if you have a goofy starting equation?
- Make it $y=mx+b$
- $3x-2y=6$ Solve for y !
- $-2y=-3x+6$ subtract $3x$ from both sides
- $y=-3/-2 x +6/-2$ divide by -2 on both sides
- $y= 3/2 x - 3$ simplify
- RIGHT OFF: $m=3/2$ and $b= -3$ or $(0,-3)$

The Standard Form

- Every line EVER created is found in the *standard form*.
- $Ax+By=C$ has every line in it!
- You just need to solve for y to find the $y=mx+b$ form.

An example (3) of going backwards TO the standard form

- Starting with $y = \frac{2}{5}x + 3$ we want $Ax + By = C$
- We are solving for the constant, 3! Weird!
- $-\frac{2}{5}x + y = 3$ I added $-\frac{2}{5}$ from both sides
- Done! But we *could* get rid of the fraction
- $5(-\frac{2}{5}x) + 5y = 5*3$ multiply everything by 5
- $-2x + 5y = 15$ So $A = -2, B = 5, C = 15$

Using $y=mx+b$ to make a graph

- So you can be given any linear equation with two variables and graph it!

Strategy for Graphing a Line Using Slope and y -Intercept

1. Write the equation in slope-intercept form if necessary.
2. Plot the y -intercept.
3. Starting from the y -intercept, use the rise and run to locate a second point.
4. Draw a line through the two points.

Example 4

- Graph $2x-3y=3$
- Solve for y
- $-3y=-2x+3$ subtract $2x$
- $y=-2/-3 x + 3/-3$ divide by -3
- $y= 2/3 x -1$ clean it up
- And you can see the $m=2/3$ and $b=-1$ which is the point $(0,-1)$

Ex 4 the graph

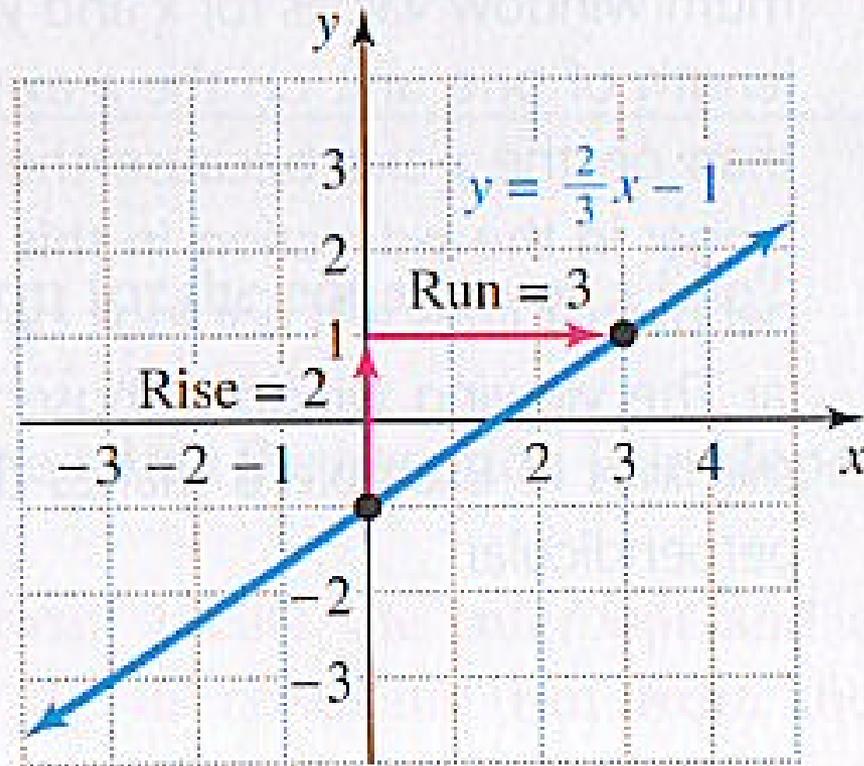


FIGURE 4.23



Another example... Ex 5

- Graph $y = -3x + 4$
 - No solving needed
 - $m = -3$
 - $b = +4$
- or $(0, 4)$

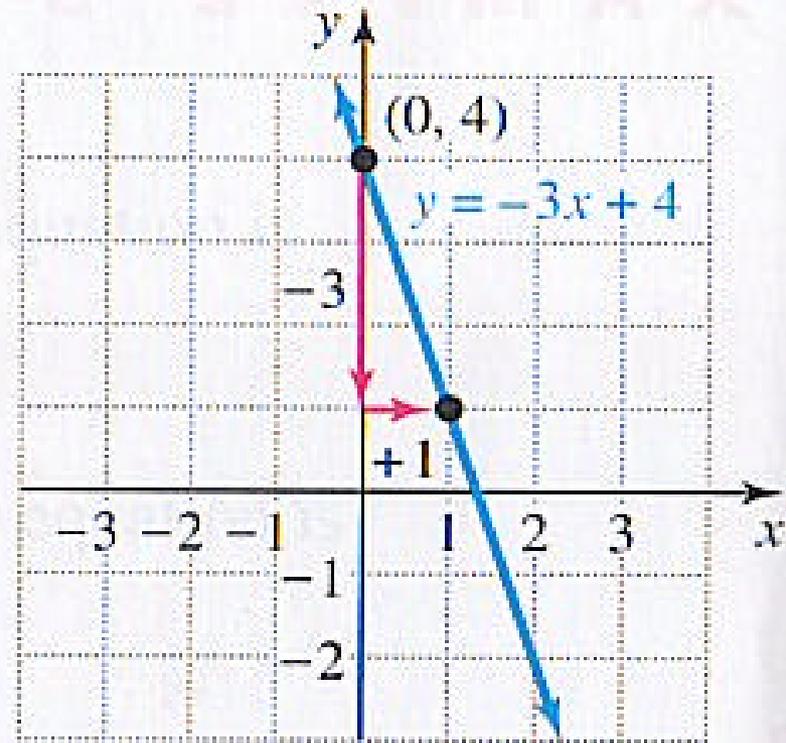


FIGURE 4.24



Goofing around with section 4.3

- Example 6: We want to write the $y=mx+b$ (the slope intercept form) of a line through $(0,4)$ (hey that's the b !) PERPENDICULAR to $2x-4y=1$
- One step at a time... what is the $y=mx+b$ form of the 'other' line?
- $-4y=-2x+1 \rightarrow y=-2/-4 x - 1/4 \rightarrow y= 1/2x- 1/4$
- So the perpendicular slope to this is $-1/m$
- $-1/(1/2) = -2$ so this is the slope we want!
- We're done! $m=-2$ and $b=4$ (given right out)
- $y= -2x +4$

Ex 7 Application

- If a landscaper has \$800 to spend on bushes which are \$20 each and trees at \$50 each. If x is the number of bushes and y is the number of trees then $20x+50y=800$. Write it in slope-intercept form ($y=mx+b$)

- $20x+50y=800$

$$50y = -20x + 800 \quad \text{subtract } 20x \text{ from both sides}$$

$$y = -20/50 x + 800/50 \quad \text{divide by } 50$$

$$y = -2/5 x + 16$$

done!

Exercises 4.3

Practice makes sore hands

- Work with $y=mx+b$!
- Definitions Q 1-6
- Write the equation from the graph Q7-18
- Find the slope and y-intercept Q19-34
- Write equations in $Ax+By=C$ Q35-50
- Draw the graphs Q51-62
- Write the $y=mx+b$ form Q63-70
- Verbal problems Q71-76

Section 4.4 The point slope form

- It's STILL $y=mx+b$ but we're peering into the 'box' a bit more
- If you have a graph, can you write $y=mx+b$?
- SURE!

Definition

- And doing this is using the point-slope form of $y=mx+b$

Point-Slope Form

The equation of the line through the point (x_1, y_1) with slope m is

$$y - y_1 = m(x - x_1).$$

For example

- If you have some line through $(4,1)$ and know the slope is $2/3$
- (shown here)

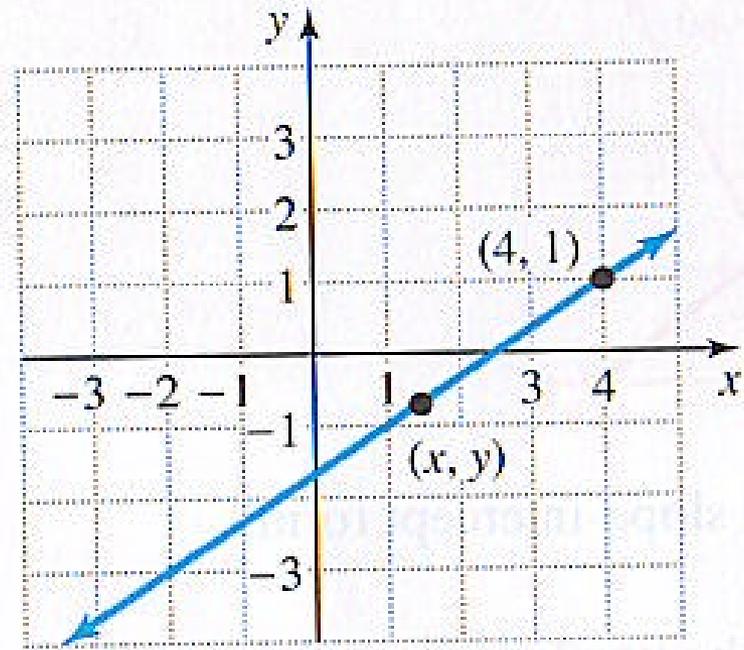


FIGURE 4.25

Example continued

- We know the slope is $(y_2 - y_1)/(x_2 - x_1) = m$
- Plug in what we know...
- $(y_2 - 1)/(x_2 - 4) = 2/3$
- $(y - 1)/(x - 4) = 2/3$ drop the subscripts- who needs 'em?
- $y - 1 = 2/3(x - 4)$ multiply both sides by $(x - 4)$
- Stop there! (we could go to $y = mx + b$ but not this time). It looks like we want it (below).

Point-Slope Form

The equation of the line through the point (x_1, y_1) with slope m is

$$y - y_1 = m(x - x_1).$$

Why are we stopping there?

- Why not take it to $y=mx+b$?
- Because it makes problems (common) where you know the slope and ANY point really easy!!!
- (With $y=mx+b$ you know the slope and JUST the y-intercept point)
- Let's see how nice it is...

Example 1

- Find the equation through $(-2,3)$ with slope $\frac{1}{2}$.
- $(-2,3)$ is NOT the intercept, it's just some point
- Use the point-slope form $y-y_1=m(x-x_1)$
- $y-3 = \frac{1}{2} [x-(-2)]$ plug in $(-2,3)$ and $\frac{1}{2}$
- $y-3 = \frac{1}{2} (x+2)$ simplify
- $y-3 = \frac{1}{2} x +1$ simplify
- $y = \frac{1}{2} x +4$ add 3 to both sides

Example 1 the old way

- If we started with the slope-intercept form (not the point-slope form) we'd do it this way...
- $y=mx+b$ **plug in $\frac{1}{2}$ for m and $(-2,3)$ for (x,y)**
- $3 = \frac{1}{2}(-2) + b$ **simplify**
- $3 = -1 + b$ **add 1 to both sides**
- $4 = b$
- Then write it out, $m = \frac{1}{2}$ (given) $b = 4$
- $y = \frac{1}{2}x + 4$

Example 2 With two points no slope

- We don't know the slope like before, but with two points $(-3,-2)$ and $(4,-1)$ we can get it...
- $m = (-2 - (-1)) / (-3 - 4) = -1 / -7 = 1/7$
- Now we are ready. With the slope and ONE point we can use the point-slope form
- next frame...

Ex 2 continued

- We found $m = 1/7$ and we can use either point – why not $(-3, -2)$ (feeling negative tonight?)
- $y - y_1 = m(x - x_1)$ **plug in the numbers**
- $y - (-2) = 1/7 [x - (-3)]$ **simplify**
- $y + 2 = 1/7(x + 3)$ **multiply by 7 to get rid of $1/7^{\text{th}}$**
- $7(y + 2) = 7[1/7(x + 3)]$ **simplify**
- $7y + 14 = x + 3$ **subtract 14 from both sides**
- $7y = x - 11$ **We want $Ax + By = C$ = standard form**
- $-x + 7y = -11$ \rightarrow $-1(-x + 7y) = -1(-11)$ **multiply by -1**
- $x - 7y = 11$ **done!**

Parallel Lines - revisited

- Example 3 Write the equation of the line parallel to the line $3x+y=9$ and contains $(2,-1)$. Give the answer in slope-intercept form ($y=mx+b$)
- First write $3x+y=9$ in slope intercept form
- $y=-3x+9$ (only had to subtract $3x$ from both sides!)
- So both lines have slope $m=-3$
- Now we're back to having a slope (-3) and finding a line through a given point $(2,-1)$. You can forget the word parallel if it makes you start to sweat.

Ex 3 continued

- $m = -3$ and point $(2, -1)$
- Use the point-slope form... it's what we have!
- $y - y_1 = m(x - x_1)$
- $y - (-1) = -3(x - 2)$ **plug in the numbers**
- $y + 1 = -3x + 6$ **simplify**
- $y = -3x + 5$ **subtract by -1 , done!**

Ex 3 visually

$$3x + y = 9$$

$$y = -3x + 9$$

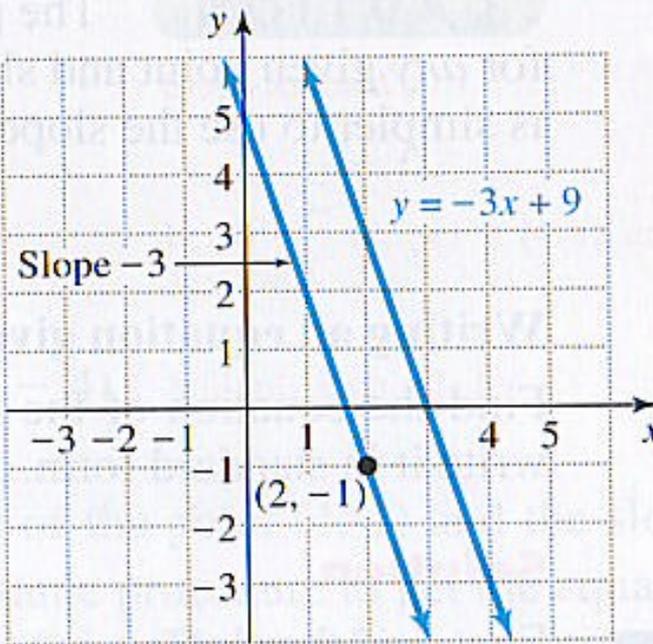


FIGURE 4.26

Perpendicular Lines - revisited

- Again, this is just a smoke and mirrors way to give us our slope. $m_1 = -(1/m_2)$
- Example 4 Find an equation perpendicular to $3x+2y=8$ and contains $(1,3)$.
- The first part gives us the slope we can flip and make negative. The second part will help us with the point-slope equation... here we go...

Example 4

- $3x+2y=8$ make it into $y=mx+b$ so we can get m
- $2y = -3x + 8$
- $y = (-3/2)x + 4$ can you see the steps?
- So $m = -3/2$ our perpendicular line will have
 $m = -(1/(-3/2)) = 2/3$ see it?

Hey! We have a slope $m=2/3$ and a point $(1,-3)$

Let's use $y-y_1=m(x-x_1)$ again!

Example 4 continued

- $y - y_1 = m(x - x_1)$ $m = 2/3$ through point $(1, -3)$
- $y - (-3) = 2/3(x - 1)$
- $y + 3 = 2/3 x - 2/3$
- $y = 2/3 x - 2/3 - 3$
- $y = 2/3 x - 2/3 - 9/3$
- $y = 2/3 x - 11/3$ Done!

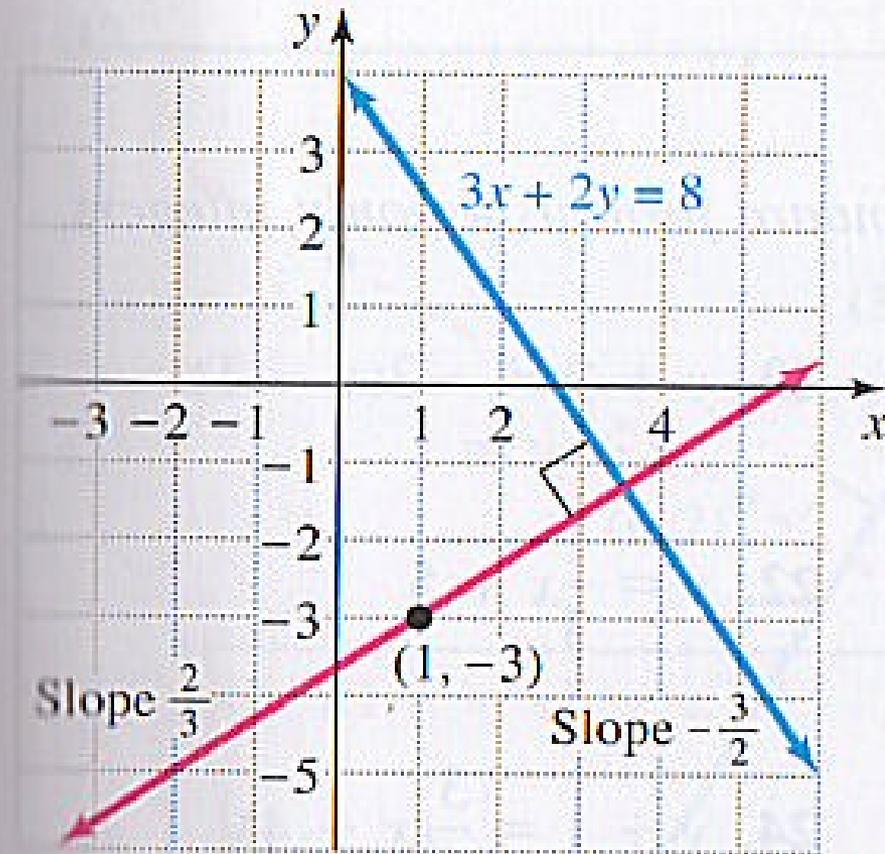


FIGURE 4.27

Section 8.6

A quantum leap in the text One small step in graphing

- Now we'll make an unholy marriage between the inequalities we worked on in Chapter 3 and the graphing of Chapter 4
- Was it EVER INTENDED to happen?
- Well, ok, sure it was.

Section 8.6

Definition

- Linear Inequalities in Two Variable
- If A,B, and C are real numbers (A&B both can't be zero) then,

$$Ax + By < C$$

Is called a *linear inequality in two variables*.

We can put \leq , $>$, or \geq in place of $<$ above as well.

The family album of linear inequalities on two variables

- $3x-4y \leq 8$
- $y > 2x-3$
- $x-y + 9 < 0$
- $x+5y \leq 22$
- Etc.

The True or False Game Returns

- Does it work?
- Example 1 With $2x-3y \geq 6$
- a) Try (4,1) \rightarrow remember it is (x,y) always

$$2(4) - 3(1) \geq? 6$$

$$8-1 \geq?6$$

$$7 \geq?6 \quad \text{NO!}$$

Ex 1b & c

$$2x - 3y \geq 6$$

b) Try (3,0)

$$2(3) - 3(0) \geq ? 6$$

$$6 \geq ? 6 \quad \text{YES!}$$

c) Try (3,-2)

$$2(3) - 3(-2) \geq ? 6$$

$$6 + 6 \geq ? 6$$

$$12 \geq ? 6 \quad \text{YES!}$$

Playing the field - again

- We just tested it point by point. That could take all day... or all year... or the rest of time to test EVERY possible point.
- Why not just draw the boundary line and shade in where it is true?
- We'll map out the entire field and have the rest of our lives to do something else!

If you have $y > x + 2$ what happens?

- So this is all places on the graph to one side or the other of $y = x + 2$ where y is larger than the line.
- For Example
(3,5) is on the line
- (3,6) is above it and true
- Note the dashed line means it is not \geq
- We'd draw it solid for \geq

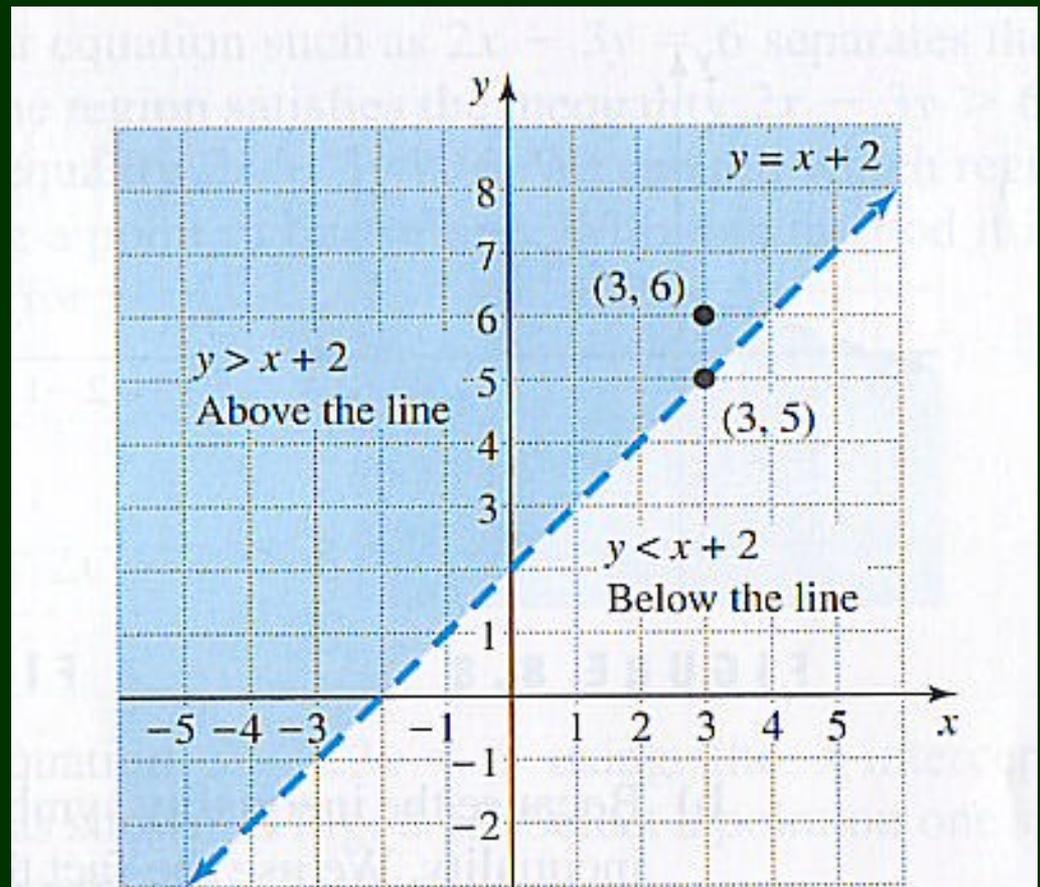


FIGURE 8.7

The cookbook

Strategy for Graphing a Linear Inequality in Two Variables

1. Solve the inequality for y , then graph $y = mx + b$.
 - $y > mx + b$ is the region above the line.
 - $y = mx + b$ is the line itself.
 - $y < mx + b$ is the region below the line.
2. If the inequality involves only x , then graph the vertical line $x = k$.
 - $x > k$ is the region to the right of the line.
 - $x = k$ is the line itself.
 - $x < k$ is the region to the left of the line.

Example 2 Doing it!

- Graph them
 - a) $y < \frac{1}{3}x + 1$

The slope is $m = \frac{1}{3}$
The intercept is $(0, 1)$
Y is “less than”
blaa blaa so shade
“below”
- Dash the line
because of $<$

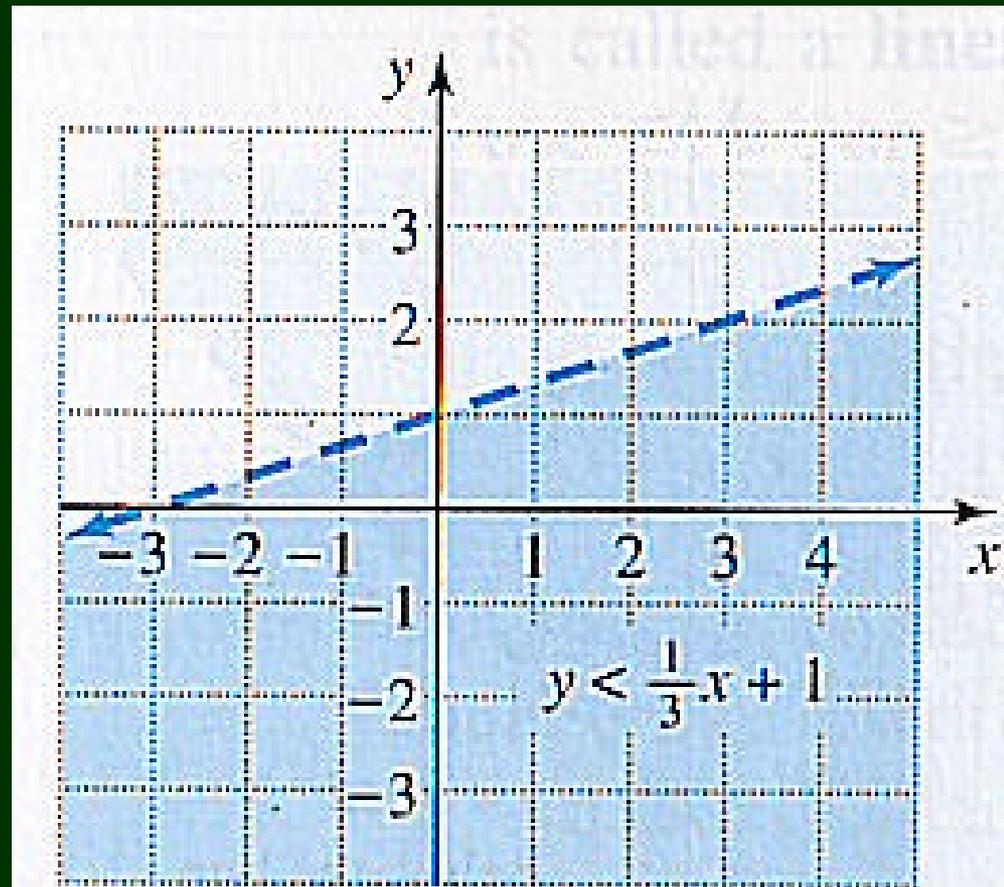


FIGURE 8.8

Example 2b

- b) $y \geq -2x + 3$
- $m = -2$
- Thru pt. $(0, 3)$
- Less than means shade below
- Equal sign means solid line not dashed

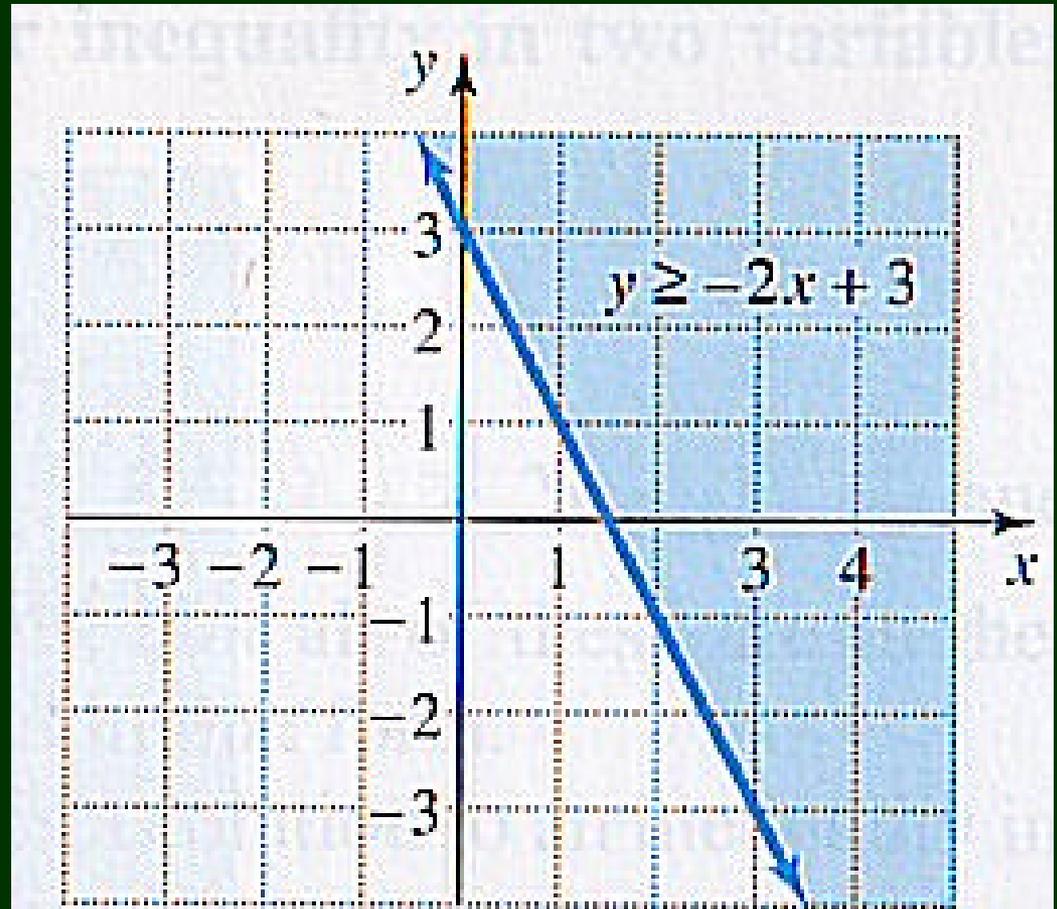
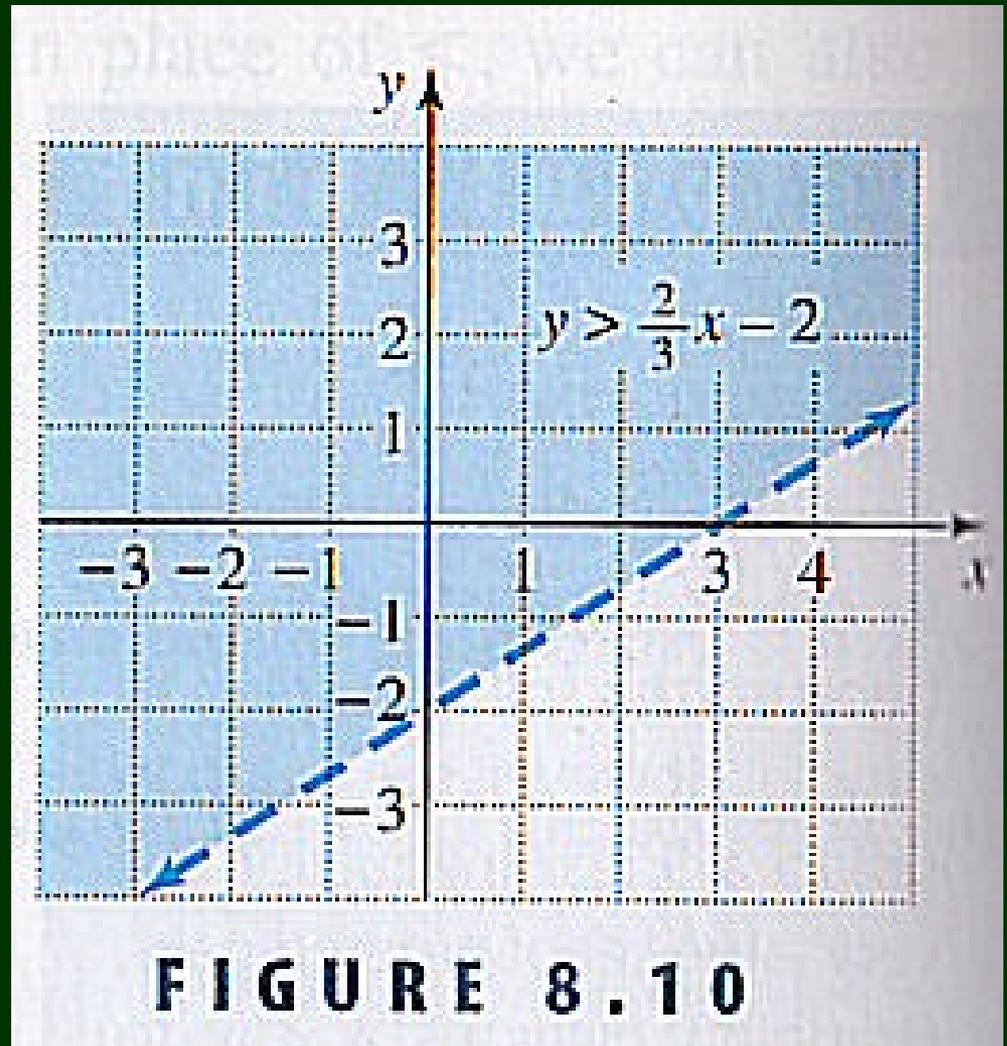


FIGURE 8.9

Example 2c

- c) $2x - 3y < 6$
Solve for y
- $-3y < -2x + 6$
 $y > \frac{2}{3}x - 2$
- $m = \frac{2}{3}$
- Pt $(0, -2)$
- y 'is greater than' so shade above
- Dashed line from $<$



Special Cases: horizontal and vertical lines (Ex 3)

- a) $y \leq 4$
 $y=4$ is a horizontal line through $(0,4)$
[there is just no x]
- Less than means shade below

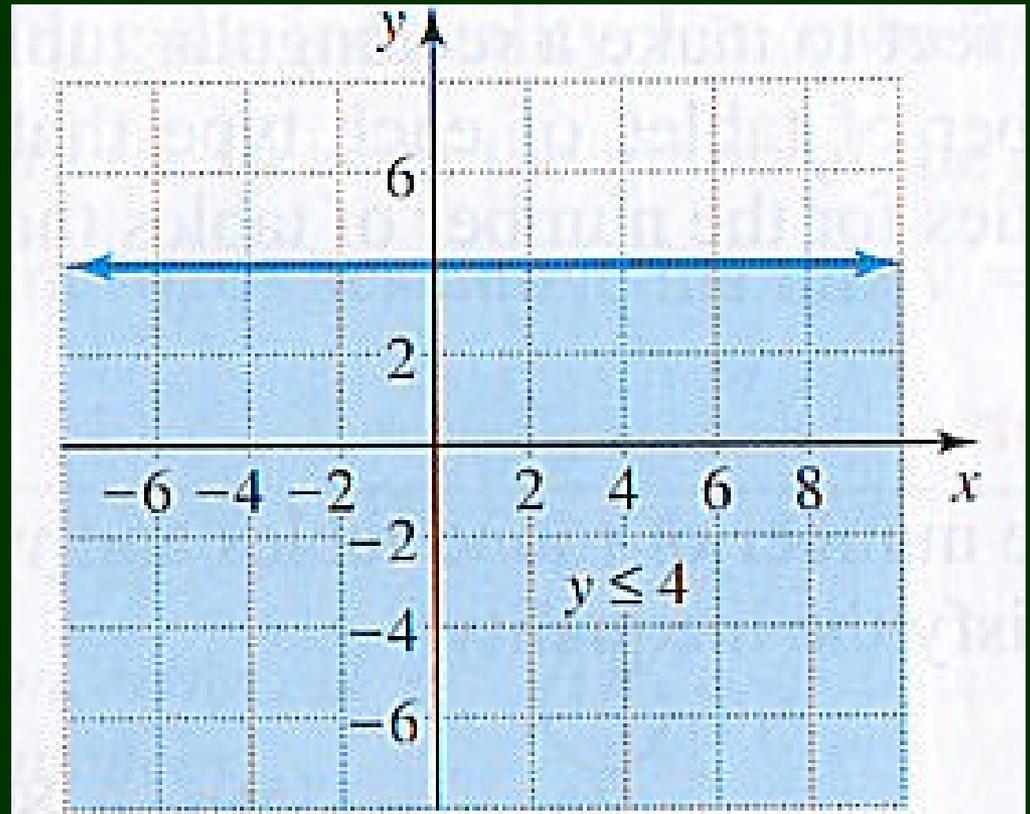


FIGURE 8.11

Ex 3b

- $x > 3$

The line is all $x=3$ points

- $>$ means dashed line

- $>$ means 'to the right'

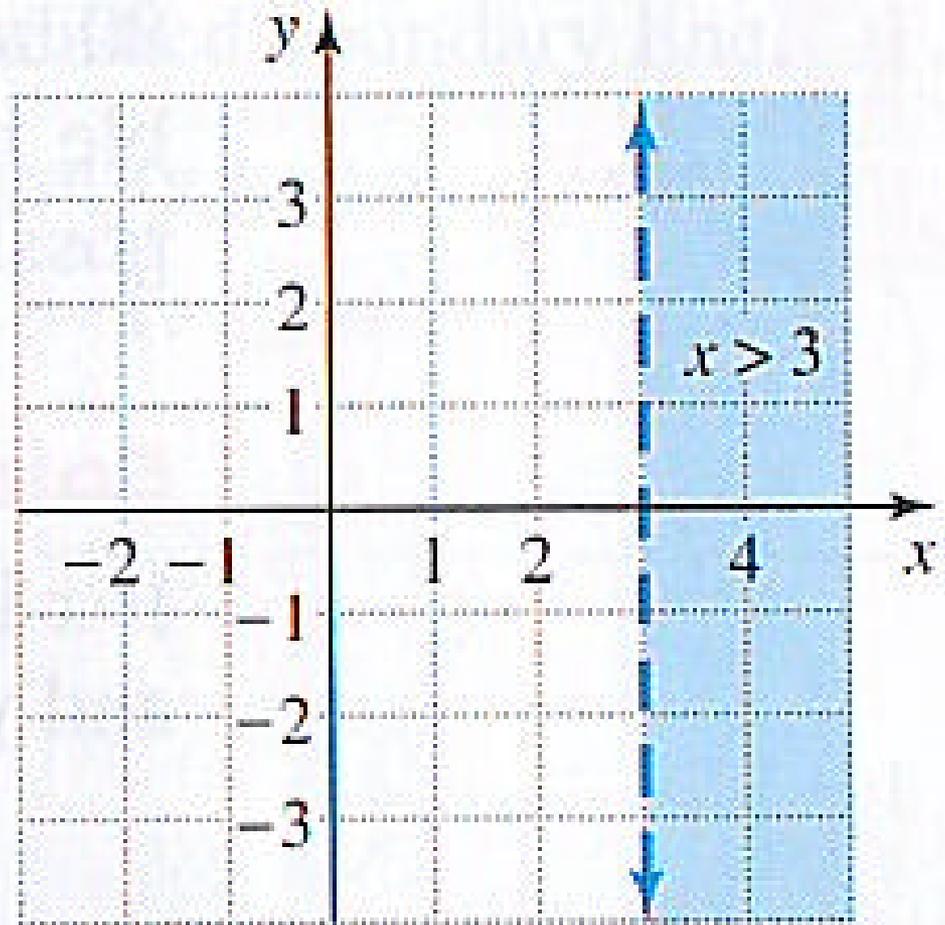


FIGURE 8.12

Confused about shading?

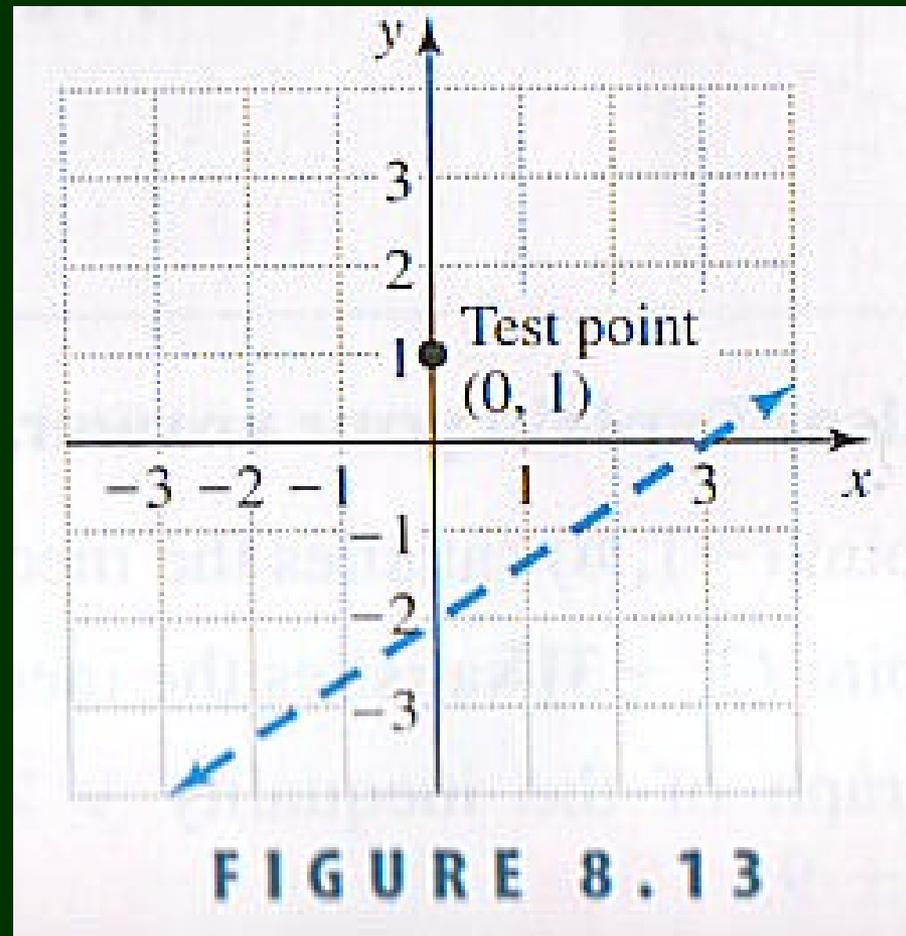
- Why not pick a test point after drawing the line (dashed or solid)?
- (You can do this until you are confident in which side the shading goes on for $>$ or $<$ inequalities).
- Remember ! Always make it look like $y > mx+b$ or $y < mx+b$ or $y \geq mx+b$ or $y \leq mx+b$

Example 4

The Test Point Shading Trick

- Graph the inequality $2x - 3y > 6$
- Solve for y
- $-3y > -2x + 6$
- $y < (-2/-3)x - 6/2$
- $y < 2/3 x - 3$
- $m = 3/2$, y -int. $(0, -3)$

- We'll test with $(0, 1)$.
- If true, we'll shade there!



Example 4 continued

- Plug out test point $(0,1)$ into $2x-3y>6$
 $2*0-3*1>6$
 $-3>6$ FALSE
- So we must shade the OTHER side...

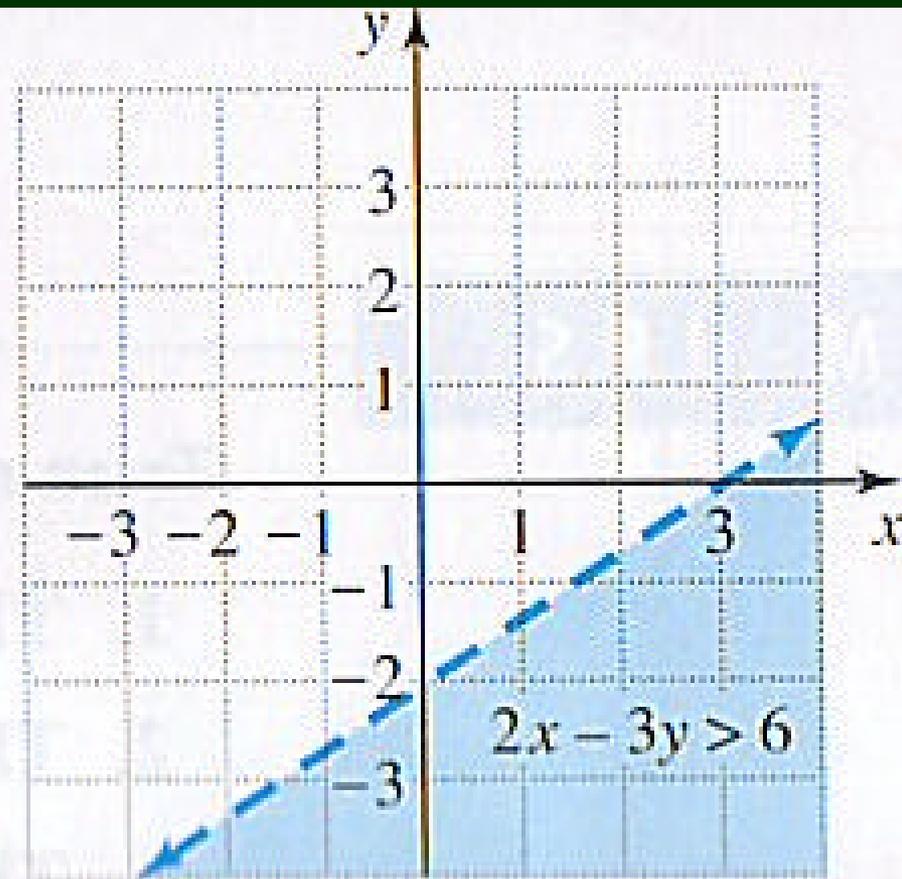


FIGURE 8.14

Another application Ex 5

- The company can obtain at MOST 8000 board feet of oak. It takes 50 board feet for round tables, 80 board feet for rectangle tables. What are all the possible combinations of round and rectangle tables they can make?
- The sum of the number of both kinds of tables is less than or equal to the 8000 max.
- $50x + 80y \leq 8000$

Ex 5 continued

- Find the intercepts (easier than solving for y and using the slope, since our units are strange and large – 100's).
- The intercepts occur where first $x=0$, then $y=0$.
- $x=0$ $50*0+80y=8000$
- $80y=8000$
- $y=100$ giving us $(0,100)$

Ex 5 goes on

- Then we find $y=0$
- $50x+80*0 = 8000$
- $50x=8000$
- $x=160$ so we have $(160,0)$

Ex 5 graphed with $(0,100)$ and $(160,0)$

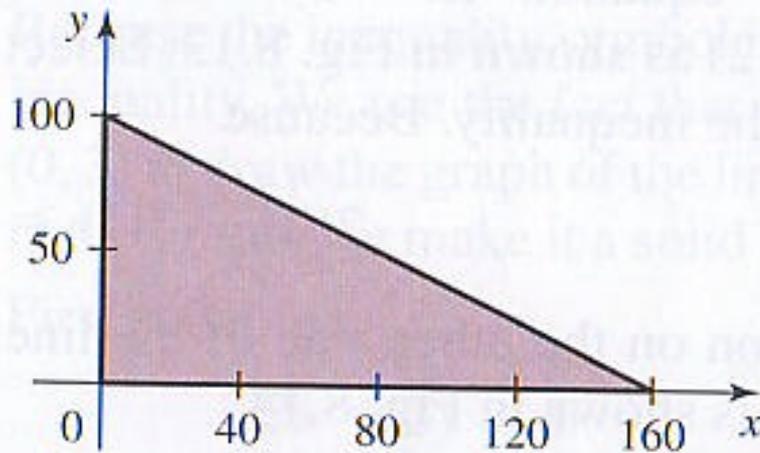


FIGURE 8.15

Exercises 8.6

- Definitions Q1-Q6
- Which points satisfy? Q7-12
- **Graph the inequality Q13-48**
- Word Problems Q49-51

Section 8.7

Two inequalities at ONCE!

- Remember when we found the intersection of two inequalities on the number line?
- We can do this with these shaded graphs as well!
- This is called solving a system of inequalities

Example 1

- We'll start out easy and just test single points again.
- The question is... does the given point satisfy both inequalities at the same time?

Example 1 continued

- $2x + 3y < 6$
 $y > 2x - 1$
- a) $(-3, 2)$ Both must be true for this to be true
- $2(-3) + 3(2) < 6$ and $-3 > 2(-3) - 1$
- $-6 + 6 < 6$ and $-3 > -6 - 1$
- $0 < 6$ TRUE and $-3 > -7$ TRUE
- This, then is a true solution to those two inequalities at the same time!

Ex 1 b

- $2x + 3y < 6$
 $y > 2x - 1$
- (4,-3) Both must be true for this to be true
- $2(4) + 3(-3) < 6$ AND $-3 > 2(4) - 1$
- $8 - 9 < 6$ AND $-3 > 8 - 1$
- $-1 < 6$ TRUE AND $-3 > 7$ FALSE
- This point is NOT a solution of these two equations.

Example 1c

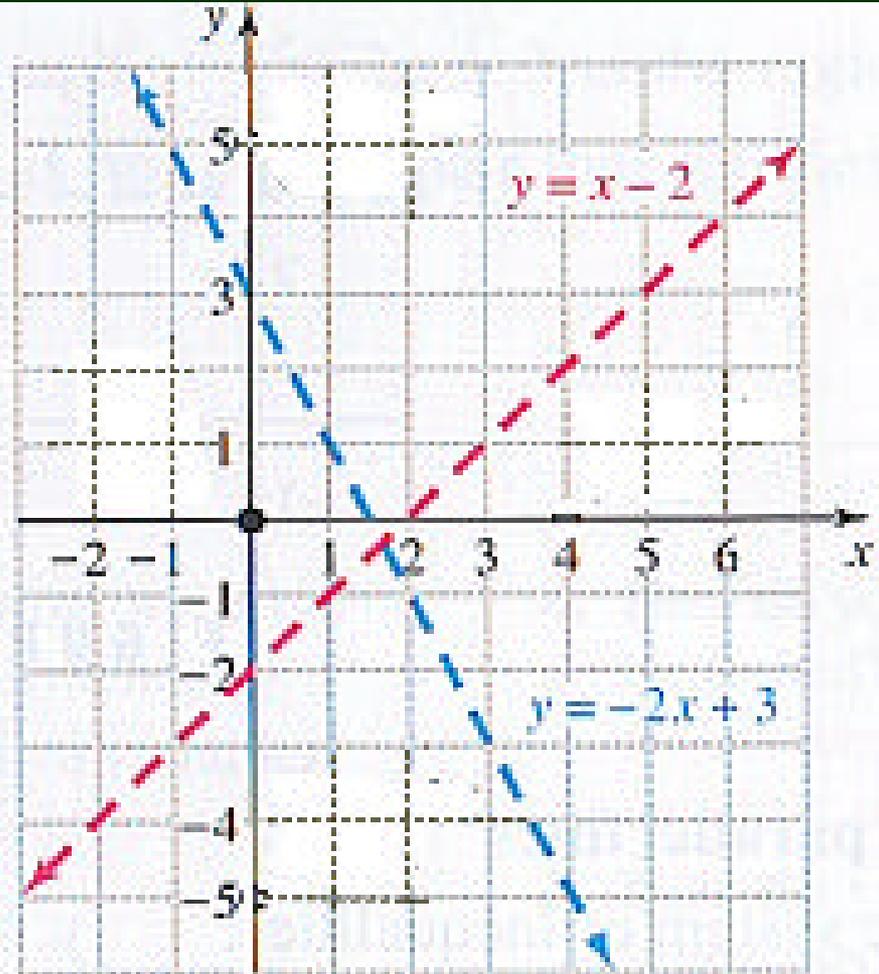
- $2x + 3y < 6$
 $y > 2x - 1$
- (5,1) Both must be true for this to be true
- $2(5) + 3(1) < 6$ AND $1 > 2(5) - 1$
- $10 + 3 < 6$ AND $1 > 10 - 1$
- $13 < 6$ FALSE AND $1 > 9$ FALSE
- This point is NOT a solution of these two equations.

Moving back to the 2-D world

- We now have two inequalities.
- That means two LINES (we can do that! It's just two lines on one graph).
- Then we only shade where the shading of both inequalities overlap.

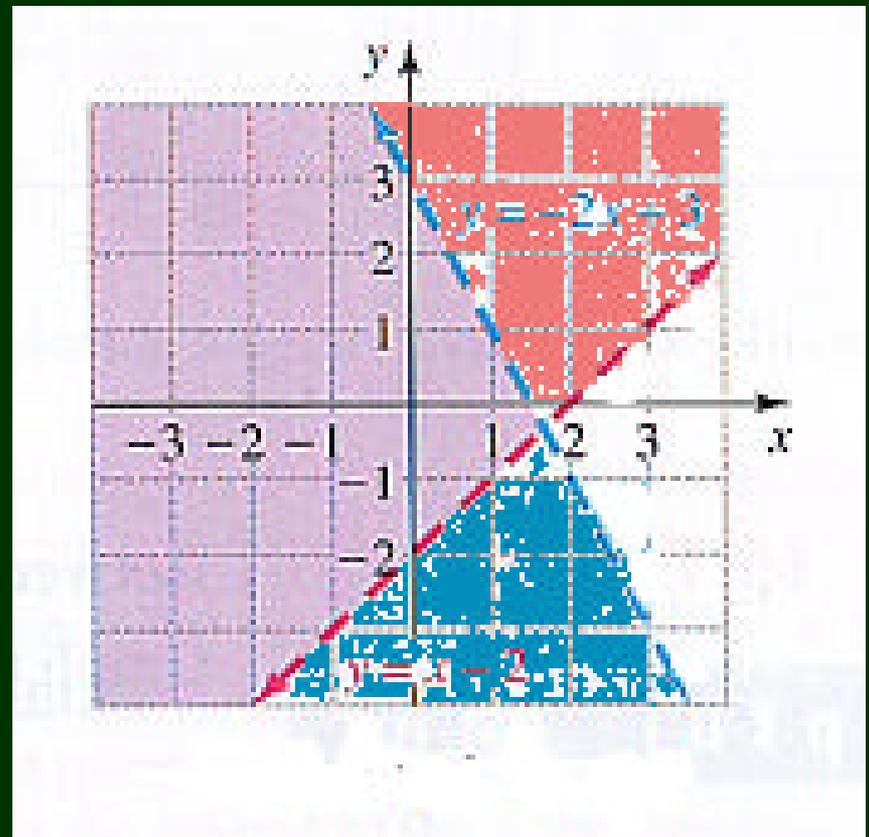
Example 2

- We are given two inequalities
- $y > x - 2$
and $y < -2x + 3$
- The lines look like this
- $y = x - 2$ and
 $y = -2x + 3$
- Dashed because of
< and >



Now to the shading

- You can lightly shade one side of each graph
 - $y > x - 2$ and $y < -2x + 3$
- Then darken the overlap



Or do the Example 1 thing: The trick of test points.

- Test a point in each quadrant, and the one that is TRUE in both is it!
- So we'll try easy ones, $(0,0)$, $(0,5)$, $(0,-5)$, $(4,0)$
- Zero is easy to work with!

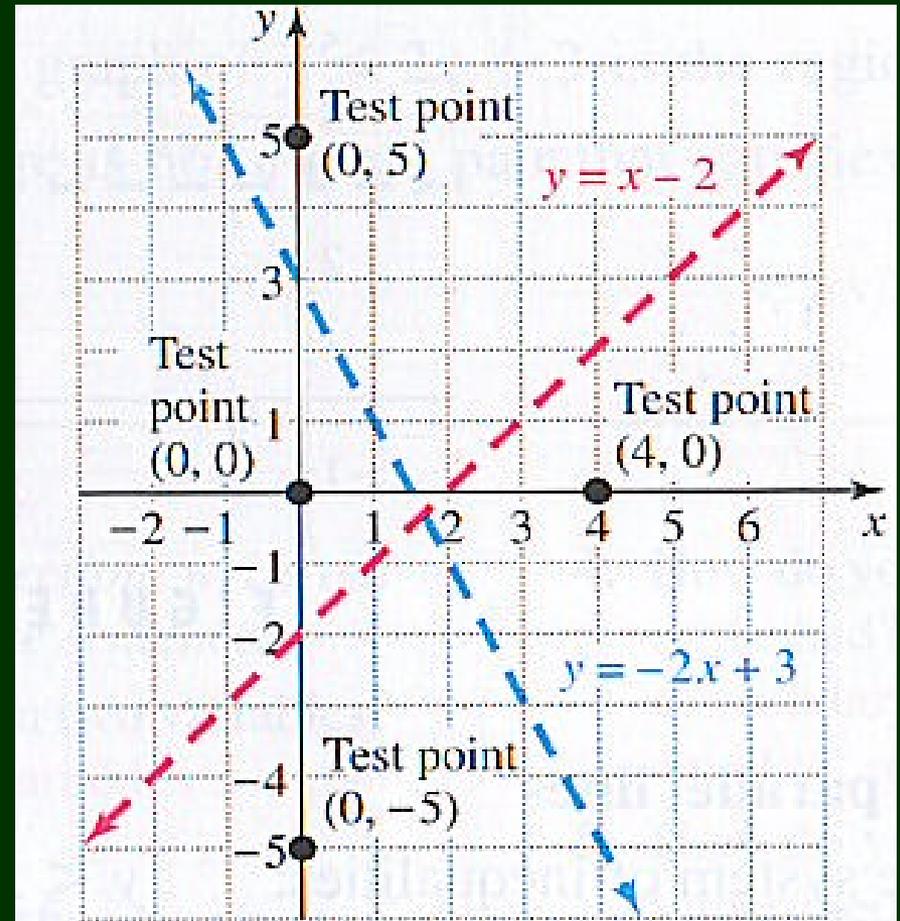


FIGURE 8.16

Ex 2 con't: Checking the points

- $y > x - 2$ and
 $y < -2x + 3$
- Check (0,0): $0 > -2$ and $0 < 3$ TRUE
- Check (0,5): $5 > -2$ and $5 < 3$ FALSE
- Check (0,-5): $-5 > -2$ and $-5 < 3$ FALSE
- Check (4,0): $0 > 2$ and $0 < -5$ FALSE
- Only ONE test point works!

Graphing using our test point

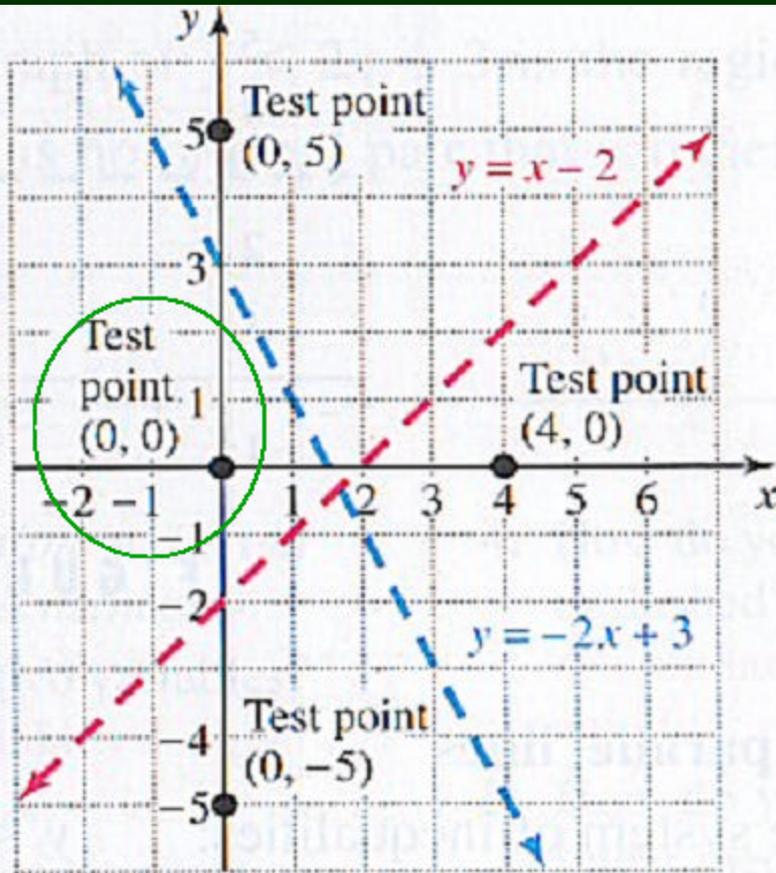


FIGURE 8.16

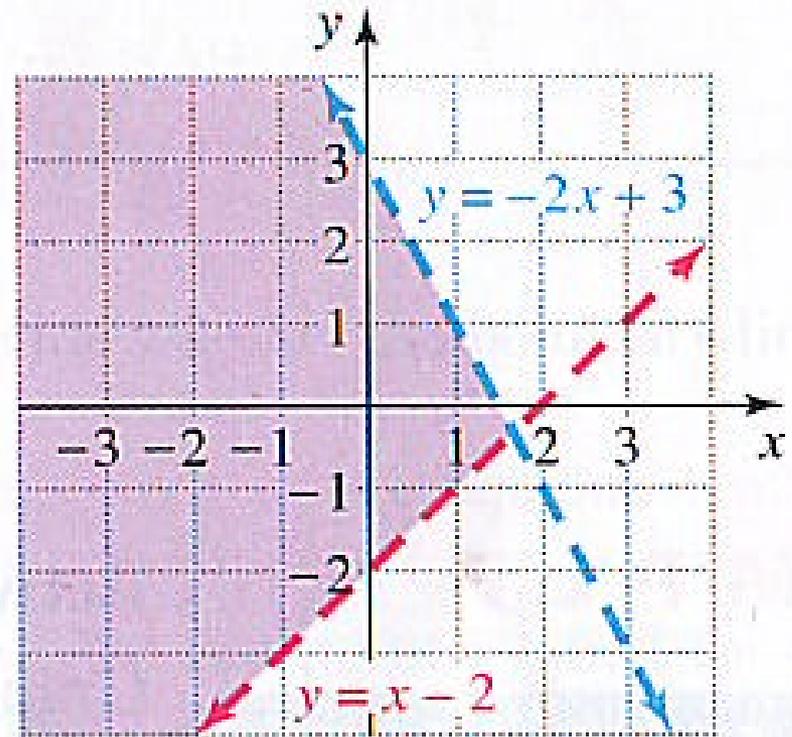


FIGURE 8.17

Example 2 Doing it Again

- Graph all *ordered pairs* (that is pairs of (x,y) solutions) that satisfy :

- $y > -3x + 4$

- $2y - x > 2$

- STEP 1! Get the $y = mx + b$ format!

- $y > -3x + 4$ is ok

$$2y - x > 2$$

$$2y > x + 2$$

$$y > \frac{1}{2}x + 1$$

Ex 3 Graphing

- $y > -3x + 4$

$$y > \frac{1}{2}x + 1$$

$$m = -3/1 \text{ and } (0, 4)$$

$$m = 1/2 \text{ and } (0, 1)$$

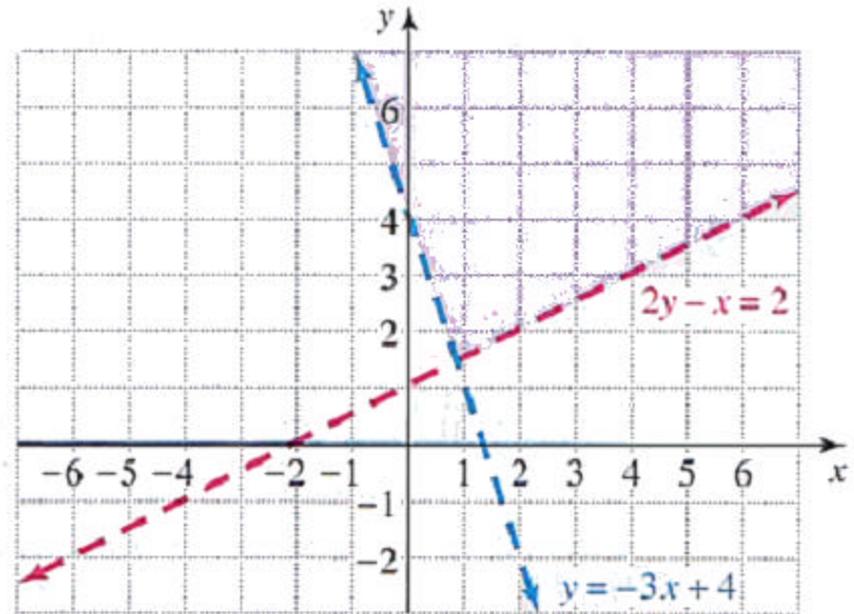


FIGURE 8.18

- Test! $(0, 0)$, $(0, 2)$, $(0, 6)$, $(5, 0)$
remember 0's are best

Ex 3 Shading?

- We'll test points in each quadrant:
- (0,0), (0,2), (0,6), (5,0)
- $y > -3x + 4$ $2y - x > 2$
- (0,0) $0 > 4$ $0 > 2$ **FALSE**
- (0,2) $2 > 4$ $4 > 2$ **FALSE**
- (0,6) $6 > 4$ $12 > 2$ **TRUE**
- (5,0) $0 > -11$ $-5 > 2$ **FALSE**

Ex 3 Shading.

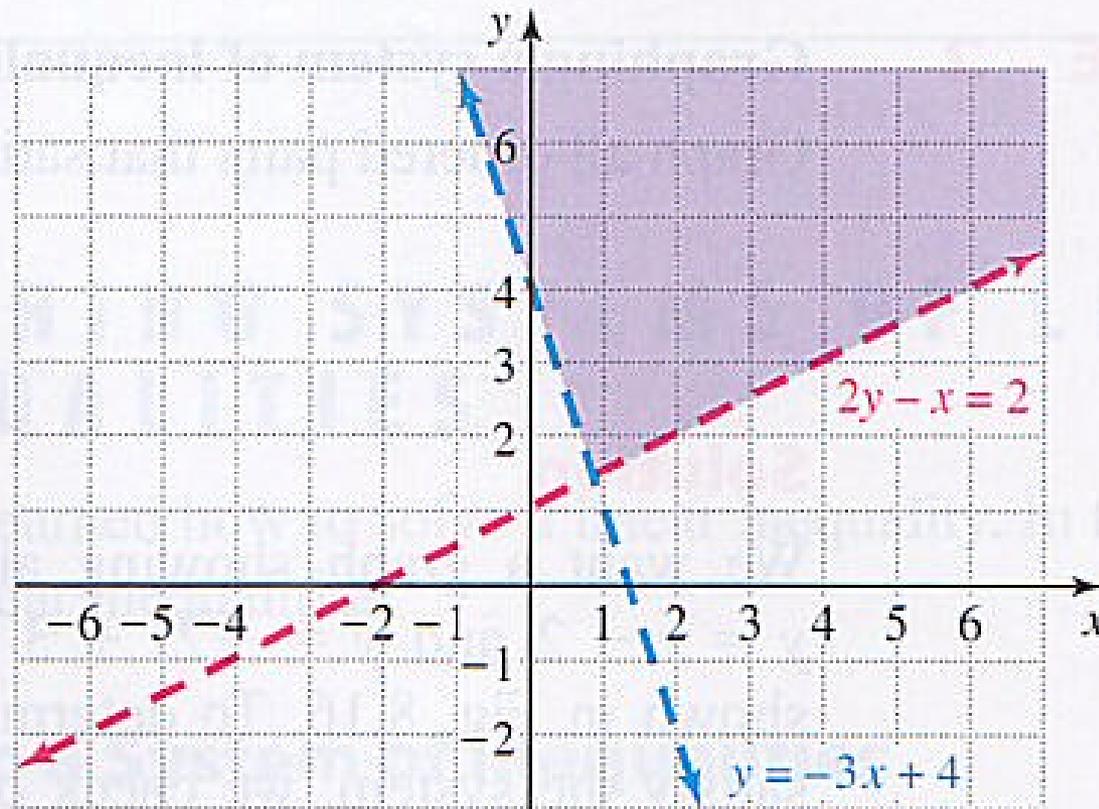


FIGURE 8.18

Ex 4 Horizontal AND Vertical Lines Revisited

- $x > 4$ and $y < 3$
- you can SEE it!

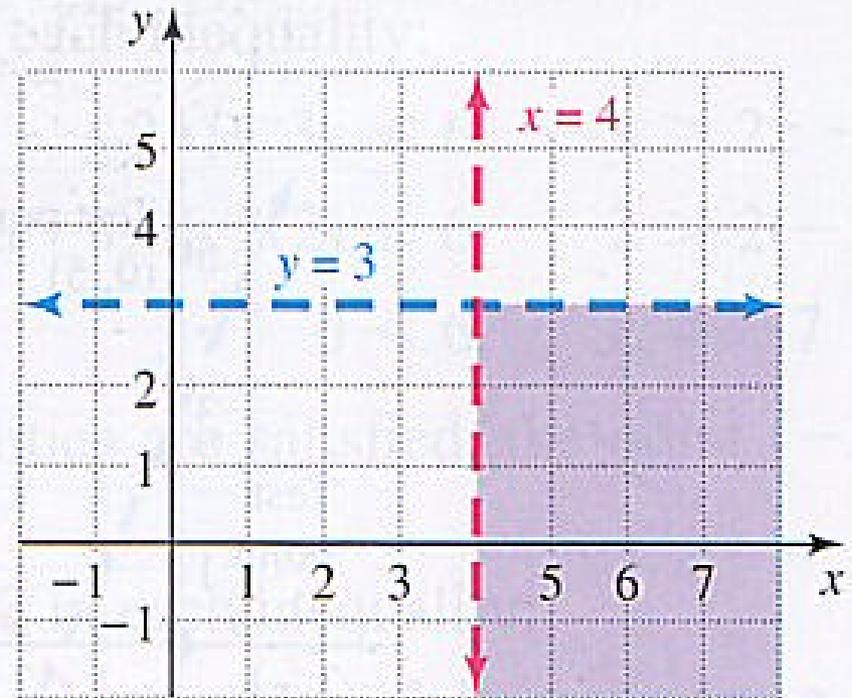


FIGURE 8.19

Ex 5 Parallel Lines

- $y < x + 4$
- $y > x - 1$
- They have the same slope!
 $m = 1$
- One goes through $(0, 4)$ the other thru $(0, -1)$

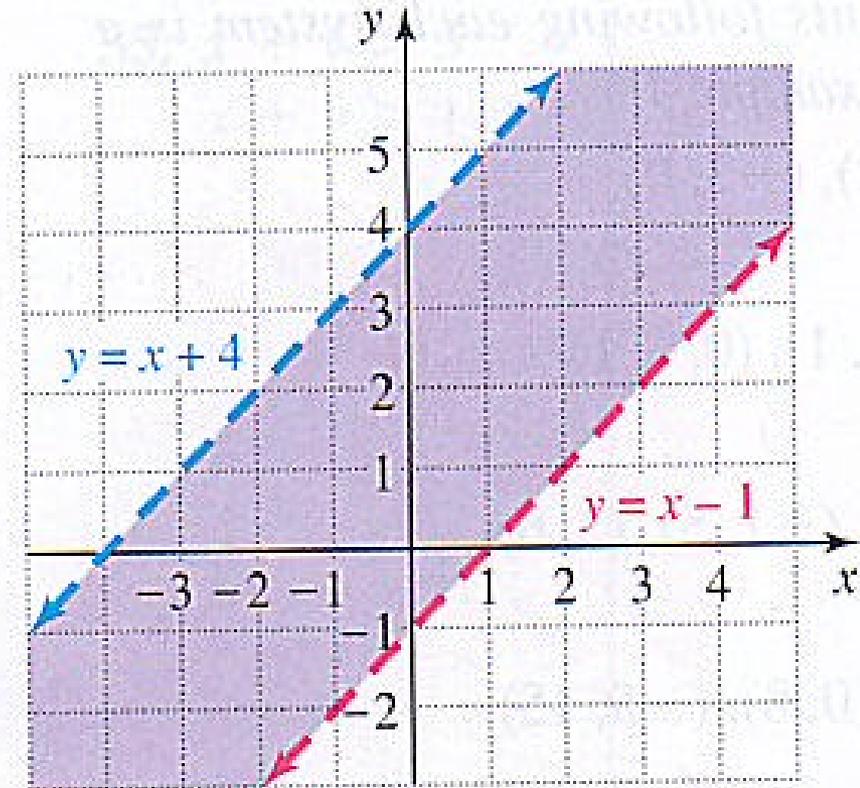


FIGURE 8.20

Exercises for section 8.7

- Shade them there graphs!
- Definitions Q1-6
- Which points solve both inequalities? Q7-12
- **Graph them! Q13-44**
- Application problems Q45-47