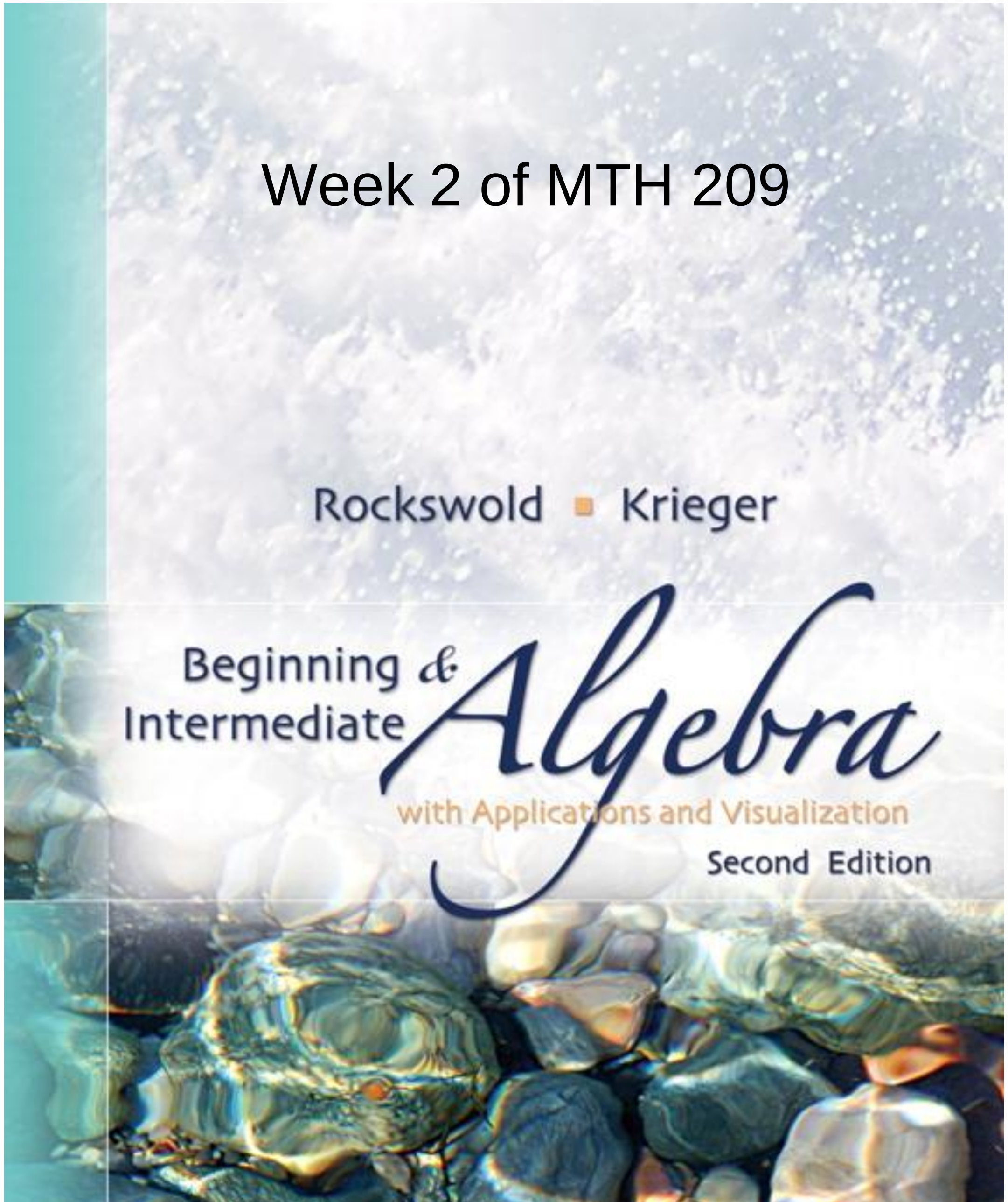
Week 2 of MTH 209



Due for this week…

- Homework 2 (on MyMathLab – via the Materials Link) → Monday night at 6pm.
- Read Chapter 6.6-6.7, 7.6-7.7, 10.5, 11.3-11.4
- Do the MyMathLab Self-Check for week 2.
- Learning team planning for week 5.
- Discuss your final week topic for your team presentations…

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

6.1 Introduction to Factoring

- Common Factors
- Factoring by Grouping

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

Common Factors

When factoring a polynomial, we first look for factors that are common to each term.

By applying a distributive property we can often write a polynomial as a product.

For example: $8x^2 = 2x \cdot 4x$ and $6x = 2x \cdot 3$

And by the distributive property,
$8x^2 + 6x = 2x(4x + 3)$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Finding common factors

Factor.

a. $6x^2+7x$ b. $15x^3-5x^2$ c. $8w^3-2w^2+4w$ d. $6x^2y^3+x^2y^2$

Solution

a. $6x^2+7x$

$$6x^2=6x\cdot x$$
$$7x=7\cdot x$$
$$x(6x+7)$$

b. $15x^3-5x^2$

$$15x^3=5\cdot 3\cdot x^2\cdot x$$
$$-5x^2=-5\cdot x^2$$
$$5x^2(3x-1)$$

c. $8w^3-2w^2+4w$

$$8w^3=4\cdot 2\cdot w^2\cdot w$$
$$-2w^2=-2\cdot w\cdot w$$
$$4w=2\cdot 2\cdot w$$
$$=2w(4w^2-w+2)$$

d. $6x^2y^3+x^2y^2$

$$6x^2y^3=6x^2y^2$$
$$x^2y^2=x^2y^2$$
$$x^2y^2(6y+1)$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

When finding the greatest common factor for a polynomial, it is often helpful first to completely factor each term of the polynomial.

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Finding the greatest common factor

Find the greatest common factor for the expression. Then factor the expression. $18x^2 + 3x$

Solution

$18x^2 = 2 \cdot 3 \cdot 3 \cdot x \cdot x$

$3x = 3 \cdot x$

The greatest common factor is $3x$.

$18x^2 + 3x = 3x(6x + 1)$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

Factoring by Grouping

Factoring by grouping is a technique that makes use of the associative and distributive properties.

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Factoring out binomials

Factor.

a. $3x(x + 1) + 4(x + 1)$ b. $3x^2(2x - 1) - x(2x - 1)$

Solution

a. Both terms in the expression contain the binomial $x + 1$. Use the distributive property to factor.

$$3x(x+1)+4(x+1)$$
$$(3x+4)(x+1)$$

b. $3x^2(2x-1)-x(2x-1)$

$$3x^2(2x-1)-x(2x-1)$$
$$(3x^2-x)(2x-1)$$
$$x(3x-1)(2x-1)$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Factoring by grouping when the middle term is (+)

Factor the polynomial. $x^3 - 4x^2 + 3x - 12$

Solution

$$x^3 - 4x^2 + 3x - 12 = (x^3 - 4x^2) + (3x - 12)$$

$$= (x^3 - 4x^2) + (3x - 12)$$

$$= x^2(x - 4) + 3(x - 4)$$

$$= (x^2 + 3)(x - 4)$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Factoring by grouping when the middle term is (–)

Factor the polynomial. $15x^3 - 10x^2 - 3x + 2$

Solution

$$
\begin{aligned}
15x^3 - 10x^2 - 3x + 2 &= (15x^3 - 10x^2) + (-3x + 2) \\
&= 5x^2(3x - 2) - 1(3x - 2) \\
&= (5x^2 - 1)(3x - 2)
\end{aligned}
$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Factoring out the GCF before grouping

Completely factor the polynomial. $30x^3 - 20x^2 - 6x + 4$

Solution

$$\begin{aligned}30x^3 - 20x^2 - 6x + 4 &= 2(15x^3 - 10x^2 - 3x + 2)\\ &= 2[(15x^3 - 10x^2) + (-3x + 2)]\\ &= 2[5x^2(3x - 2) - 1(3x - 2)]\\ &= 2(5x^2 - 1)(3x - 2)\end{aligned}$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

Practice for section 6.1

- Finding common factors questions 13-20
- Finding the greatest common factor Q 21-38
- Factoring out binomials Q 39-44
- Factoring by grouping when the middle term is (+) Q 45-48
- Factoring by grouping when the middle term is (–) Q 53-59
- Factoring out the GCF before grouping Q 65-70

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

6.2 Factoring Trinomials I ($x^2 + bx + c$)

- Review of the FOIL Method
- Factoring Trinomials Having a Leading Coefficient of 1

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

The product $(x + 3)(x + 4)$ can be found as follows:

$$x^2 + 4x + 3x + 12$$

$$x^2 + 7x + 12$$

The middle term is found by calculating the sum $4x$ and $3x$, and the last term is found by calculating the product of 4 and 3.

FACTORING $x^2 + bx + c$

To factor the trinomial $x^2 + bx + c$, find numbers m and n that satisfy

$$m \cdot n = c \quad \text{and} \quad m + n = b.$$

Then $x^2 + bx + c = (x + m)(x + n)$.

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE

Factoring a trinomial having only positive coefficients

Factor each trinomial.

a. $x^2 + 11x + 18$

b. $x^2 + 10x + 24$

Solution

a. $x^2 + 11x + 18$

Factors of 18 whose sum is 11.

Factors	Sum
1, 18	19
2, 9	**11**
3, 6	9

$$(x+2)(x+9)$$

b. $x^2 + 10x + 24$

Factors of 24 whose sum is 10.

Factors	Sum
1, 24	25
2, 12	14
3, 8	11
4, 6	**10**

$$(x+4)(x+6)$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Factoring a trinomial having a negative middle coefficient

Factor each trinomial.

a. $x^2 - 9x + 14$

b. $x^2 - 19x + 48$

Solution

a. $x^2 - 9x + 14$

Factors of 14 whose sum is −9.

Factors	Sum
$-1, -14$	-15
$-2, -7$	**-9**

$$(x - 2)(x - 7)$$

b. $x^2 - 19x + 48$

Factors of 48 whose sum is −19.

Factors	Sum
$-1, -48$	-49
$-2, -24$	-26
$-3, -16$	**-19**
$-4, -12$	-16
$-6, -8$	-14

$$(x - 3)(x - 16)$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Factoring a trinomial having a negative constant term

Factor each trinomial.

a. $x^2 + 2x - 15$ b. $x^2 - 6x - 16$

Solution

a. $x^2 + 2x - 15$

Factors of −15 whose sum is 2.

Factors	Sum
1, −15	−14
− 1, 15	+14
3, −5	−2
−3, 5	**2**

$(x-3)(x+5)$

b. $x^2 - 6x - 16$

Factors of −16 whose sum is −6.

Factors	Sum
−1, 16	15
1, −16	−15
−2, 8	6
2, −8	**−6**
−4, 4	0

$(x+2)(x-8)$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Discovering that a trinomial is prime

Factor the trinomial. $x^2 + 6x - 8$

Solution

$x^2 + 6x - 8$

Factors of −8 whose sum is 6.

Factors	Sum
1, −8	−7
−1, 8	7
2, −4	−2
−2, 4	2

The table reveals that no such factor pair exists. Therefore, the trinomial is prime.

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Factoring out the GCF before factoring further

Factor each trinomial completely.

a. $4x^2 + 28x + 48$ b. $7x^2 + 21x - 70$

Solution

a. $4x^2 + 28x + 48$

Factor out common factor of 4.

$4(x^2 + 7x + 12)$

Factors (12)	Sum (7)
1, 12	13
2, 6	8
3, 4	**7**

$4(x+3)(x+4)$

b. $7x^2 + 21x - 70$

Factor out common factor of 7.

$7(x^2 + 3x - 10)$

Factors (–10)	Sum (3)
1, –10	–9
2, –5	–3
–2, 5	**3**

$7(x-2)(x+5)$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Finding the dimensions of a rectangle

Find one possibility for the dimensions of a rectangle that has an area of $x^2 + 12x + 35$.

Solution

	x	5
x	x^2	$5x$
7	$7x$	35

Area = $x^2 + 12x + 35$

The area of a rectangle equals length times width. If we can factor $x^2 + 12x + 35$, then the factors can represents its length and width.

Because

$$x^2 + 12x + 35 = (x + 5)(x + 7),$$

one possibility for the rectangle's dimensions is width $x + 5$ and length $x + 7$.

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

Practice for section 6.2

- Factoring a trinomial having only positive coefficients *Q 15-26*
- Factoring a trinomial having a negative middle coefficient *Q 29-38*
- Factoring a trinomial having a negative constant term *Q 39-60*
- Factoring out the GCF before factoring further Q 61-80
- Finding the dimensions of a rectangle Q 83

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

6.3

Factoring Trinomials II ($ax^2 + bx + c$)

- Factoring Trinomials by Grouping
- Factoring with FOIL in Reverse

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

Factoring Trinomials by Grouping

FACTORING $ax^2 + bx + c$ BY GROUPING

To factor $ax^2 + bx + c$ perform the following steps. (Assume that a, b, and c have no factor in common.)

1. Find numbers m and n such that $mn = ac$ and $m + n = b$. (This step may require trial and error.)
2. Write the trinomial as $ax^2 + mx + nx + c$.
3. Use grouping to factor this expression as two binomials.

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Factoring $ax^2 + bx + c$ by grouping

Factor each trinomial.

a. $2x^2 + 13x + 15$

b. $12y^2 - 5y - 3$

Solution

a. $2x^2 + 13x + 15$

Multiply (2)(15) = 30
Factors of 30 whose sum is 13
10 and 3

$$= 2x^2 + 10x + 3x + 15$$
$$= (2x^2 + 10x) + (3x + 15)$$
$$= 2x(x + 5) + 3(x + 5)$$
$$= (2x + 3)(x + 5)$$

b. $12y^2 - 5y - 3$

Multiply (12)(–3) = –36
Factors of –36 whose sum is –5
– 9 and 4

$$= 12y^2 - 9y + 4y - 3$$
$$= (12y^2 - 9y) + (4y - 3)$$
$$= 3y(4y - 3) + 1(4y - 3)$$
$$= (3y + 1)(4y - 3)$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Discovering that a trinomial is prime

Factor the trinomial. $3x^2+9x+4$

Solution

a. We need to find integers *m* and *n* such that $mn = (3)(4) = 12$ and $m + n = 9$. Because the middle term is positive, we consider only positive factors of 12.

Factors	1, 12	2, 6	3, 4
Sum	13	8	7

There are no factors whose sum is 9, the coefficient of the middle term. The trinomial is prime.

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

Factoring with FOIL in Reverse

$$3x^2 + 7x + 2 \stackrel{?}{=} (\underline{\quad\quad} + \underline{\quad\quad})(\underline{\quad\quad} + \underline{\quad\quad})$$

$$\mathbf{3x^2} + 7x + 2 \stackrel{?}{=} (\underline{\;\mathbf{3x}\;} + \underline{\quad\quad})(\underline{\;\mathbf{x}\;} + \underline{\quad\quad}).$$

$$(3x + 2)(x + 1) = 3x^2 + 5x + 2$$

$$\begin{array}{r} 2x \\ +3x \\ \hline 5x \end{array} \longleftarrow \text{Middle term is } \textit{not } 7x.$$

$$(\mathbf{3x} + \mathbf{1})(\mathbf{x} + \mathbf{2}) = 3x^2 + 7x + 2$$

$$\begin{array}{r} 1x \\ +6x \\ \hline 7x \end{array} \longleftarrow \text{Middle term checks.}$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Factoring the form $ax^2 + bx + c$

Factor the trinomial. $2x^2 + 7x + 6$

Solution

$$2x^2 + 7x + 6$$

$$2x^2 + 7x + 6 = (2x + \_\_\_)(x + \_\_\_)$$

The factors of the last term are either 1 and 6 or 2 and 3.
Try a set of factors. Try 1 and 6.

$$(2x + 1)(x + 6) = 2x^2 + 13x + 6$$

$$\begin{array}{r} x \\ \underline{12x} \\ 13x \end{array}$$

Middle term is 13*x* not 7*x*.

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Factoring the form $ax^2 + bx + c$—*continued*

Factor the trinomial. $2x^2 + 7x + 6$

The factors of the last term are either 1 and 6 or 2 and 3.

Try a set of factors.

Solution

Try 2 and 3 the factors of the last term.

$$(2x+2)(x+3) = 2x^2 + 8x + 6$$

$$\begin{array}{r} 2x \\ \underline{6x} \\ 8x \end{array}$$

Middle term is 8*x* not 7*x*.

Try another set of factors 3 and 2.

$$(2x+3)(x+2) = 2x^2 + 7x + 6$$

$$\begin{array}{r} 3x \\ \underline{4x} \\ 7x \end{array}$$

Middle term is correct.

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

MAKING CONNECTIONS

The Signs in the Binomial Factors

Let a, b, and c represent positive integers. If a trinomial of the form $ax^2 + bx + c$ can be factored, the signs in the binomial factors can be summarized as follows.

Form of the Trinomial	*Signs in the Binomial Factors*
$ax^2 + bx + c$	$(\ \ +\ \ )(\ \ +\ \ )$
$ax^2 - bx + c$	$(\ \ -\ \ )(\ \ -\ \ )$
$ax^2 + bx - c$	$(\ \ -\ \ )(\ \ +\ \ )$
$ax^2 - bx - c$	$(\ \ -\ \ )(\ \ +\ \ )$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

Practice for section 6.3

- Factoring $ax^2 + bx + c$ by grouping Q11-34
- Discovering that a trinomial is prime Q 13-23
- Factoring the form $ax^2 + bx + c$ Q 29-50

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

6.4 Special Types of Factoring

- Difference of Two Squares
- Perfect Square Trinomials
- Sum and Difference of Two Cubes

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

DIFFERENCE OF TWO SQUARES

For any real numbers a and b,

$$a^2 - b^2 = (a - b)(a + b).$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Factoring the difference of two squares

Factor each difference of two squares.

a. $9x^2 - 16$ b. $5x^2 + 8y^2$ c. $25x^4 - y^6$

Solution

a. $9x^2 - 16 = (3x)^2 - (4)^2 = (3x - 4)(3x + 4)$

b. Because $5x^2 + 8y^2$ is the sum of two squares, it *cannot* be factored.

c. If we let $a^2 = 25x^4$ and $b^2 = y^6$, then $a = 5x^2$ and $b = y^3$. Thus,

$$25x^4 - y^6 = (5x^2)^2 - (y^3)^2$$
$$= (5x^2 - y^3)(5x^2 + y^3).$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

PERFECT SQUARE TRINOMIALS

For any real numbers a and b,

$$a^2 + 2ab + b^2 = (a + b)^2 \quad \text{and}$$

$$a^2 - 2ab + b^2 = (a - b)^2.$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Factoring perfect square trinomials

If possible, factor each trinomial as a perfect square.

a. $x^2 + 8x + 16$

b. $4x^2 - 12x + 9$

Solution

a. $x^2 + 8x + 16$ Let $a^2 = x^2$ and $b^2 = 4^2$. For a perfect square trinomial, the middle term must be $2ab$.

$$2ab = 2(x)(4) = 8x,$$

which equals the given middle term. Thus $a^2 + 2ab + b^2 = (a + b)^2$ implies

$$x^2 + 8x + 16 = (x + 4)^2.$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE continued

b. $4x^2 - 12x + 9$

Let $a^2 = (2x)^2$ and $b^2 = 3^2$. For a perfect square trinomial, the middle term must be $2ab$.

$$2ab = 2(2x)(3) = 12x,$$

which equals the given middle term. Thus $a^2 - 2ab + b^2 = (a - b)^2$ implies

$$4x^2 - 12x + 9 = (2x - 3)^2.$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

SUM AND DIFFERENCE OF TWO CUBES

For any real numbers *a* and *b*,

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2) \quad \text{and}$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE

Factoring the sum and difference of two cubes

Factor each polynomial.

a. $n^3 + 27$ b. $8x^3 - 125y^3$

Solution

a. $n^3 + 27$

Because $n^3 = (n)^3$ and $27 = 3^3$, we let $a = n$, $b = 3$, and factor.

Substituting $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

gives $n^3 + 3^3 = (n + 3)(n^2 - n \cdot 3 + 3^2)$

$= (n + 3)(n^2 - 3n + 9)$.

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE continued

b. $8x^3 - 125y^3$

Here, $8x^3 = (2x)^3$ and $125y^3 = (5y)^3$, so

$$8x^3 - 125y^3 = (2x)^3 - (5y)^3.$$

Substituting $a = 2x$ and $b = 5y$ in

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

gives

$$(2x)^3 - (5y)^3 = (2x - 5y)(4x^2 + 10xy + 25y^2).$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE

Factoring out the GCF before factoring further

Factor the polynomial completely. $8s^3 - 32st^2$

Solution

Factor out the common factor of $8s$.

$$8s^3 - 32st^2 = 8s(s^2 - 4t^2)$$
$$= 8s(s - 2t)(s + 2t)$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

Practice for section 6.4

- Factoring the difference of two squares Q 15-30
- Factoring perfect square trinomials Q 31-52
- Factoring the sum and difference of two cubes Q53-66
- Factoring out the GCF before factoring further Q 67-84

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

7.1 Introduction to Rational Expressions

- Basic Concepts
- Simplifying Rational Expressions
- Applications

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

Basic Concepts

Rational expressions can be written as quotients (fractions) of two polynomials.

Examples include:

$$\frac{5}{x}, \qquad \frac{x^2}{3x-4}, \qquad \frac{4x^2+6x-1}{4x^3-8}$$

RATIONAL EXPRESSION

A rational expression can be written as $\frac{P}{Q}$, where P and Q are polynomials. A rational expression is defined whenever $Q \neq 0$.

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Evaluating rational expressions

If possible, evaluate each expression for the given value of the variable.

a. $\dfrac{1}{x+3}; x=3$ b. $\dfrac{w^2}{3w-4}; w=4$ c. $\dfrac{4-w}{w-4}; w=-5$

Solution

a. $\dfrac{1}{x+3}; x=3$

$$\frac{1}{3+3}=\frac{1}{6}$$

b. $\dfrac{w^2}{3w-4}; w=4$

$$=\frac{(4)^2}{3(4)-4}$$

$$=\frac{16}{12-4}$$

$$=\frac{16}{8}=2$$

c. $\dfrac{4-w}{w-4}; w=-5$

$$=\frac{4-(-5)}{(-5)-4}$$

$$=\frac{9}{-9}=-1$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Determining when a rational expression is undefined

Find all values of the variable for which each expression is undefined.

a. $\frac{1}{x^2}$ b. $\frac{w^2}{w-4}$ c. $\frac{6}{w^2-4}$

Solution

a. $\frac{1}{x^2}$ b. $\frac{w^2}{w-4}$ c. $\frac{6}{w^2-4}$

Undefined when $x^2 = 0$ or when $x = 0$.

Undefined when $w - 4 = 0$ or when $w = 4$.

Undefined when $w^2 - 4 = 0$ or when $w = \pm 2$.

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Simplifying fractions

Simplify each fraction by applying the basic principle of fractions.

a. $\dfrac{9}{15}$ b. $\dfrac{20}{28}$ c. $\dfrac{45}{135}$

Solution

a. The GCF of 9 and 15 is 3. $\dfrac{9}{15} = \dfrac{3 \cdot 3}{3 \cdot 5} = \dfrac{3}{5}$

b. The GCF of 20 and 28 is 4. $\dfrac{20}{28} = \dfrac{4 \cdot 5}{4 \cdot 7} = \dfrac{5}{7}$

c.The GCF of 45 and 135 is 45. $\dfrac{45}{135} = \dfrac{45 \cdot 1}{45 \cdot 3} = \dfrac{1}{3}$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

BASIC PRINCIPLE OF RATIONAL EXPRESSIONS

The following property can be used to simplify rational expressions, where P, Q, and R are polynomials.

$$\frac{P \cdot R}{Q \cdot R} = \frac{P}{Q} \qquad Q \text{ and } R \text{ are nonzero.}$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Simplifying rational expressions

Simplify each expression.

a. $\dfrac{16y}{4y^2}$ b. $\dfrac{3x+12}{4x+16}$ c. $\dfrac{x^2-25}{2x^2-7x-15}$

Solution

a. $\dfrac{16y}{4y^2}$

$$=\frac{4y\cdot 4}{4y\cdot y}$$

$$=\frac{4}{y}$$

b. $\dfrac{3x+12}{4x+16}$

$$=\frac{3(x+4)}{4(x+4)}$$

$$=\frac{3}{4}$$

c. $\dfrac{x^2-25}{2x^2-7x-15}$

$$=\frac{(x-5)(x+5)}{(2x+3)(x-5)}$$

$$=\frac{x+5}{2x+3}$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Distributing a negative sign

Simplify each expression.

a. $\dfrac{-y-7}{2y+14}$

b. $-\dfrac{8-x}{x-8}$

Solution

a. $\dfrac{-y-7}{2y+14}$

$$= \frac{-1(y+7)}{2(y+7)}$$

$$= -\frac{1}{2}$$

b. $-\dfrac{8-x}{x-8}$

$$= \frac{-(8-x)}{x-8}$$

$$= \frac{-8+x}{x-8}$$

$$= \frac{x-8}{x-8} = 1$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

Practice for section 7.1

- Evaluating rational expressions Q 11-24
- Determining when a rational expression is undefined Q 29-42
- Simplifying fractions Q 43-50
- Simplifying rational expressions Q 55-66,71-84
- Distributing a negative sign Q 67-70

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

7.2 Multiplication and Division of Rational Expressions

- Review of Multiplication and Division of Fractions
- Multiplication of Rational Expressions
- Division of Rational Expressions

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Multiplying fractions

Multiply and simplify your answers to lowest terms.

a. $\dfrac{4}{9} \cdot \dfrac{5}{7}$ b. $15 \cdot \dfrac{4}{5}$ c. $\dfrac{2}{7} \cdot \dfrac{5}{8}$

Solution

a. $\dfrac{4}{9} \cdot \dfrac{5}{7} = \dfrac{20}{63}$

b. $15 \cdot \dfrac{4}{5} = \dfrac{15}{1} \cdot \dfrac{4}{5} = \dfrac{60}{5} = 12$

c. $\dfrac{2}{7} \cdot \dfrac{5}{8} = \dfrac{5 \cdot 2}{7 \cdot 8} = \dfrac{5}{7} \cdot \dfrac{1}{4} = \dfrac{5}{28}$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Dividing fractions

Divide and simplify your answers to lowest terms.

a. $\frac{1}{6} \div \frac{3}{5}$ b. $\frac{6}{7} \div 18$ c. $\frac{4}{5} \div \frac{11}{15}$

Solution

a. $$\frac{1}{6} \div \frac{3}{5} = \frac{1}{6} \cdot \frac{5}{3} = \frac{5}{18}$$

b. $$\frac{6}{7} \div 18 = \frac{6}{7} \cdot \frac{1}{18} = \frac{1 \cdot 6}{7 \cdot 18} = \frac{1}{21}$$

c. $$\frac{4}{5} \div \frac{11}{15} = \frac{4}{5} \cdot \frac{15}{11} = \frac{60}{55} = \frac{12 \cdot 5}{11 \cdot 5} = \frac{12}{11}$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

PRODUCTS OF RATIONAL EXPRESSIONS

To multiply two rational expressions multiply the numerators and multiply the denominators. That is,

$$\frac{A}{B} \cdot \frac{C}{D} = \frac{AC}{BD},$$

where B and D are nonzero.

EXAMPLE Multiplying rational expressions

Multiply and simplify to lowest terms. Leave your answers in factored form.

a. $\frac{6x}{10} \cdot \frac{5}{12x^2}$

b. $\frac{x-3}{2x-1} \cdot \frac{x+4}{3x-9}$

Solution

a. $\frac{6x}{10} \cdot \frac{5}{12x^2} = \frac{6x}{10} \cdot \frac{5}{12x^2}$

$= \frac{30x}{120x^2}$

$= \frac{1}{4x}$

b. $\frac{x-3}{2x-1} \cdot \frac{x+4}{3x-9}$

$= \frac{(x-3)(x+4)}{(2x-1)(3x-9)}$

$= \frac{(x-3)(x+4)}{3(2x-1)(x-3)}$

$= \frac{x+4}{3(2x-1)}$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Multiplying rational expressions

Multiply and simplify to lowest terms. Leave your answer in factored form.

$$\frac{x^2-16}{x^2-9}\cdot\frac{x+3}{x-4}$$

Solution

$$\frac{x^2-16}{x^2-9}\cdot\frac{x+3}{x-4}=\frac{(x^2-16)(x+3)}{(x^2-9)(x-4)}$$

$$=\frac{(x-4)(x+4)(x+3)}{(x-3)(x+3)(x-4)}$$

$$=\frac{(x+4)(x+3)(x-4)}{(x-3)(x+3)(x-4)}$$

$$=\frac{(x+4)}{(x-3)}$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

QUOTIENTS OF RATIONAL EXPRESSIONS

To divide two rational expressions multiply by the reciprocal of the divisor. That is,

$$\frac{A}{B} \div \frac{C}{D} = \frac{A}{B} \cdot \frac{D}{C},$$

where B, C, and D are nonzero.

EXAMPLE Dividing rational expressions

Divide and simplify to lowest terms.

a. $\dfrac{3}{x} \div \dfrac{2x+1}{6x}$

b. $\dfrac{x^2-16}{x^2-2x-8} \div \dfrac{x+4}{x+2}$

Solution

a. $\dfrac{3}{x} \div \dfrac{2x+1}{6x}$

$$= \frac{3}{x} \cdot \frac{6x}{2x+1}$$

$$= \frac{18x}{x(2x+1)}$$

$$= \frac{18}{2x+1}$$

b. $\dfrac{x^2-16}{x^2-2x-8} \div \dfrac{x+4}{x+2}$

$$= \frac{x^2-16}{x^2-2x-8} \cdot \frac{x+2}{x+4}$$

$$= \frac{(x+4)(x-4)}{(x+2)(x-4)} \cdot \frac{x+2}{x+4}$$

$$= \frac{(x+4)(x-4)(x+2)}{(x+2)(x-4)(x+4)} \quad = 1$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

Practice for section 7.2

- Multiplying fractions Q 9-14
- Dividing fractions Q 17-22
- Multiplying rational expressions Q 33-49
- Dividing rational expressions Q 53-70

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

7.3 Addition and Subtraction With Like Denominators

- Review of Addition and Subtraction of Fractions
- Rational Expressions Having Like Denominators

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Adding fractions having like denominators

Add and simplify to lowest terms.

a. $\frac{4}{7}+\frac{1}{7}$

b. $\frac{1}{9}+\frac{5}{9}$

Solution

a. $\frac{4}{7}+\frac{1}{7} = \frac{4+1}{7} = \frac{5}{7}$

b. $\frac{1}{9}+\frac{5}{9} = \frac{1+5}{9} = \frac{6}{9} = \frac{2}{3}$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Subtracting fractions having like denominators

Subtract and simplify to lowest terms.

a. $\dfrac{13}{18}-\dfrac{7}{18}$ b. $\dfrac{15}{30}-\dfrac{11}{30}$

Solution

a.
$$\frac{13}{18}-\frac{7}{18}=\frac{13-7}{18}=\frac{6}{18}=\frac{1}{3}$$

b.
$$\frac{15}{30}-\frac{11}{30}=\frac{15-11}{30}=\frac{4}{30}=\frac{2}{15}$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

SUMS OF RATIONAL EXPRESSIONS

To add two rational expressions having like denominators, add their numerators. Keep the same denominator.

$$\frac{A}{C} + \frac{B}{C} = \frac{A+B}{C}$$

C is nonzero

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE **Adding rational expressions having like denominators**

Add and simplify.

a. $\dfrac{4x+1}{x+3}+\dfrac{x-2}{x+3}$ b. $\dfrac{x}{x^2+7x+10}+\dfrac{5}{x^2+7x+10}$

Solution

a. $$\frac{4x+1}{x+3}+\frac{x-2}{x+3}=\frac{4x+1+x-2}{x+3}=\frac{5x-1}{x+3}$$

b. $$\frac{x}{x^2+7x+10}+\frac{5}{x^2+7x+10}=\frac{x+5}{x^2+7x+10}$$

$$=\frac{x+5}{(x+5)(x+2)}=\frac{1}{x+2}$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE **Adding rational expressions having two variables**

Add and simplify to lowest terms.

a. $\dfrac{7}{ab}+\dfrac{4}{ab}$ b. $\dfrac{w}{w^2-y^2}+\dfrac{y}{w^2-y^2}$

Solution

a. $$\frac{7}{ab}+\frac{4}{ab} = \frac{7+4}{ab} = \frac{11}{ab}$$

b. $$\frac{w}{w^2-y^2}+\frac{y}{w^2-y^2} = \frac{w+y}{w^2-y^2} = \frac{w+y}{(w-y)(w+y)} = \frac{1}{w-y}$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

DIFFERENCES OF RATIONAL EXPRESSIONS

To subtract two rational expressions having like denominators, subtract their numerators. Keep the same denominator.

$$\frac{A}{C} - \frac{B}{C} = \frac{A - B}{C}$$

C is nonzero

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE

Subtracting rational expressions having like denominators

Subtract and simplify to lowest terms.

a. $\dfrac{6}{x^2}-\dfrac{x+6}{x^2}$ b. $\dfrac{2x-3}{x^2-1}-\dfrac{x-4}{x^2-1}$

Solution

a. $\dfrac{6}{x^2}-\dfrac{x+6}{x^2}=\dfrac{6-(x+6)}{x^2}=\dfrac{6-x-6}{x^2}=\dfrac{-x}{x^2}=-\dfrac{1}{x}$

b. $\dfrac{2x-3}{x^2-1}-\dfrac{x-4}{x^2-1}=\dfrac{2x-3-x+4}{x^2-1}=\dfrac{x+1}{x^2-1}$

$$=\frac{x+1}{(x+1)(x-1)}=\frac{1}{x-1}$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE **Subtracting rational expressions having like denominators**

Subtract and simplify to lowest terms. $\dfrac{7a}{a+2}-\dfrac{a-2}{a+2}$

Solution

$$\frac{7a}{a+2}-\frac{a-2}{a+2}=\frac{7a-(a-2)}{a+2}=\frac{6a+2}{a+2}$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

Practice for section 7.3

- Adding fractions having like denominators Q 15-20
- Subtracting fractions having like denominators Q 27-34
- Adding rational expressions having like denominators Q 25, 39, 41
- Adding rational expressions having two variables Q 57-62
- Subtracting rational expressions having like denominators Q 37, 63

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

7.4 Addition and Subtraction with Unlike Denominators

- Finding Least Common Multiples
- Review of Fractions Having Unlike Denominators
- Rational Expressions Having Unlike Denominators

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

FINDING THE LEAST COMMON MULTIPLE

The least common multiple (LCM) of two or more polynomials can be found as follows.

Step 1: Factor each polynomial completely.

Step 2: List each factor the greatest number of times that it occurs in either factorization.

Step 3: Find the product of this list of factors. The result is the LCM.

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Finding least common multiples

Find the least common multiple of each pair of expressions.

a. $6x, 9x^4$ b. $x^2 + 7x + 12, x^2 + 8x + 16$

Solution

a. *Step 1*: Factor each polynomial completely.

$$6x = 3 \cdot 2 \cdot x \qquad 9x^4 = 3 \cdot 3 \cdot x \cdot x \cdot x \cdot x$$

Step 2: List each factor the greatest number of times.

$$3 \cdot 3 \cdot 2 \cdot x \cdot x \cdot x \cdot x$$

Step 3: The LCM is $18x^4$.

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE continued

Find the least common multiple.

b. $x^2 + 7x + 12, x^2 + 8x + 16$

Step 1: Factor each polynomial completely.

$$x^2 + 7x + 12 = (x + 3)(x + 4)$$

$$x^2 + 8x + 16 = (x + 4)(x + 4)$$

Step 2: List each factor the greatest number of times.

$(x + 3)$, $(x + 4)$, and $(x + 4)$

Step 3: The LCM is $(x + 3)(x + 4)^2$.

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE

Adding and subtracting fractions having unlike denominators

Simplify each expression.

a. $\frac{4}{7}+\frac{1}{6}$ b. $\frac{5}{12}-\frac{11}{30}$

Solution

a. The LCD is the LCM, 42.

$$\frac{4}{7}+\frac{1}{6}=\frac{4}{7}\cdot\frac{6}{6}+\frac{1}{6}\cdot\frac{7}{7}=\frac{24}{42}+\frac{7}{42}=\frac{31}{42}$$

b. The LCD is 60.

$$\frac{5}{12}-\frac{11}{30}=\frac{5}{12}\cdot\frac{5}{5}-\frac{11}{30}\cdot\frac{2}{2}=\frac{25}{60}-\frac{22}{60}=\frac{3}{60}=\frac{1}{20}$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE

Adding rational expressions having unlike denominators

Find each sum and leave your answer in factored form.

a. $\dfrac{2}{x}+\dfrac{5}{x^2}$

b. $\dfrac{4}{x-1}+\dfrac{3}{1-x}$

Solution

a. $$\frac{2}{x}+\frac{5}{x^2} = \frac{2}{x}\cdot\frac{x}{x}+\frac{5}{x^2} = \frac{2x}{x^2}+\frac{5}{x^2} = \frac{2x+5}{x^2}$$

b. $$\frac{4}{x-1}+\frac{3}{1-x} = \frac{4}{x-1}+\frac{3}{1-x}\cdot\frac{-1}{-1} = \frac{4}{x-1}+\frac{-3}{x-1} = \frac{1}{x-1}$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Subtracting rational expressions having unlike denominators

Simply each expression. Write your answer in lowest terms and leave it in factored form. $\frac{x-3}{x}-\frac{5}{x+7}$

Solution

The LCD is $x(x + 7)$.

$$\frac{x-3}{x}-\frac{5}{x+7}=\frac{x-3}{x}\cdot\frac{x+7}{x+7}-\frac{5}{x+7}\cdot\frac{x}{x}=\frac{(x-3)(x+7)}{x(x+7)}-\frac{5x}{x(x+7)}$$

$$=\frac{(x-3)(x+7)-5x}{x(x+7)}=\frac{x^2+4x-21-5x}{x(x+7)}=\frac{x^2-x-21}{x(x+7)}$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Subtracting rational expressions with unlike denominators

Simplify each expression. Write your answer in lowest terms and leave it in factored form.

$$\frac{6}{x^2+6x+9}-\frac{5}{x^2-9}=\frac{6}{(x+3)(x+3)}-\frac{5}{(x+3)(x-3)}$$

The LCD is (x + 3)(x + 3)(x – 3).

$$=\frac{6}{(x+3)(x+3)}\cdot\frac{(x-3)}{(x-3)}-\frac{5}{(x+3)(x-3)}\cdot\frac{(x+3)}{(x+3)}$$

$$=\frac{6(x-3)}{(x+3)(x+3)(x-3)}-\frac{5(x+3)}{(x+3)(x+3)(x-3)}$$

$$=\frac{6x-18-5x-15}{(x+3)(x+3)(x-3)}\qquad=\frac{x-33}{(x+3)(x+3)(x-3)}$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Modeling electrical resistance

Add $\frac{1}{R}+\frac{1}{S}$, and then find the reciprocal of the result.

Solution

The LCD is *RS*.

$$\frac{1}{R}+\frac{1}{S}=\frac{1}{R}\cdot\frac{S}{S}+\frac{1}{S}\cdot\frac{R}{R}$$

$$=\frac{S}{RS}+\frac{R}{RS}$$

$$=\frac{S+R}{RS}$$

The reciprocal is

$$\frac{RS}{S+R}.$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

Practice for section 7.4

- Finding least common multiples Q 15-38
- Adding and subtracting fractions having unlike denominators Q 51-58
- Adding rational expressions having unlike denominators Q 59-77
- Subtracting rational expressions having unlike denominators Q 69-87
- Modeling electrical resistance Q 103

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

10.1 Radical Expressions and Rational Exponents

- Radical Notation
- Rational Exponents
- Properties of Rational Exponents

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

SQUARE ROOT

The number b is a *square root* of a if $b^2 = a$.

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

Every positive number *a* has two square roots, one positive and one negative. Recall that the *positive* square root is called the *principal square root.*

The symbol $\sqrt{}$ is called the **radical sign.**

The expression under the radical sign is called the **radicand**, and an expression containing a radical sign is called a **radical expression**.

Examples of radical expressions:

$$\sqrt{7}, \quad 6+\sqrt{x+2}, \text{ and } \sqrt{\frac{5x}{3x-4}}$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Finding principal square roots

Evaluate each square root.

a. $\sqrt{36} = 6$

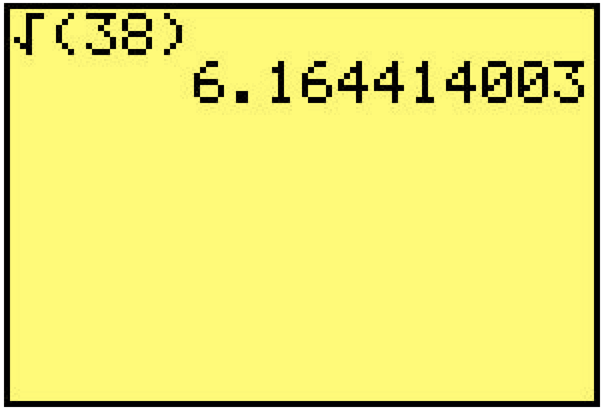
b. $\sqrt{0.64} = 0.8$

c. $\sqrt{\dfrac{16}{25}} = \dfrac{4}{5}$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Approximating a square root

Approximate $\sqrt{38}$ to the nearest thousandth.



$$= 6.164$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

CUBE ROOT

The number b is a *cube root* of a if $b^3 = a$.

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

THE NOTATION $\sqrt[n]{a}$

The equation $\sqrt[n]{a} = b$ means that $b^n = a$, where n is a natural number called the **index**. If n is odd, we are finding an **odd root** and if n is even, we are finding an **even root**.

1. If $a > 0$, then $\sqrt[n]{a}$ is a positive number.

2. If $a < 0$ and

   a. n is odd, then $\sqrt[n]{a}$ is a negative number.

   b. n is even, then $\sqrt[n]{a}$ is *not* a real number.

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Finding nth roots

Find each root, if possible.

a. $\sqrt[4]{256}$ b. $\sqrt[5]{-243}$ c. $\sqrt[4]{-1296}$

Solution

a. $\sqrt[4]{256} = 4$ because $4 \cdot 4 \cdot 4 \cdot 4 = 256.$

b. $\sqrt[5]{-243} = -3$ because $(-3)^5 = -243.$

c. $\sqrt[4]{-1296}$ An *even* root of a *negative* number is *not* a real number.

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

THE EXPRESSION $\sqrt{x^2}$

For every real number x, $\sqrt{x^2} = |x|$.

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Simplifying expressions

Write each expression in terms of an absolute value.

a. $\sqrt{(-5)^2}$ b. $\sqrt{(x+3)^2}$ c. $\sqrt{w^2-6w+9}$

Solution

a. $\sqrt{(-5)^2} = |-5| = 5$

b. $\sqrt{(x+3)^2} = |x+3|$

c. $\sqrt{w^2-6w+9} = \sqrt{(w-3)^2} = |w-3|$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

THE EXPRESSION $a^{1/n}$

If n is an integer greater than 1 and a is a real number, then

$$a^{1/n} = \sqrt[n]{a}.$$

NOTE: If $a < 0$ and n is an even positive integer, then $a^{1/n}$ is not a real number.

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Interpreting rational exponents

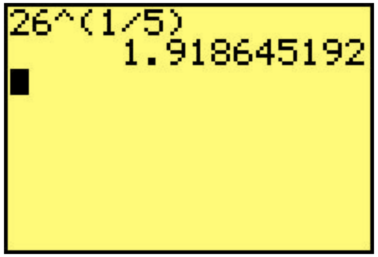
Write each expression in radical notation. Then evaluate the expression and round to the nearest hundredth when appropriate.

a. $49^{1/2}$ b. $26^{1/5}$ c. $(6x)^{1/2}$

Solution

a. $49^{1/2} = \sqrt{49} = 7$

b. $26^{1/5}$



c. $(6x)^{1/2} = \sqrt{6x}$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

THE EXPRESSION $a^{m/n}$

If m and n are positive integers with $\frac{m}{n}$ in lowest terms, then

$$a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m.$$

NOTE: If $a < 0$ and n is an even integer, then $a^{m/n}$ is *not* a real number.

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Interpreting rational exponents

Write each expression in radical notation. Evaluate the expression by hand when possible.

a. $81^{3/4}$

b. $14^{4/5}$

Solution

a. $81^{3/4}$ Take the fourth root of 81 and then cube it.

$$= (81)^{3/4}$$

$$= \left(\sqrt[4]{81}\right)^{3}$$

$$= 3^{3}$$

$$= 27$$

b. $14^{4/5}$ Take the fifth root of 14 and then fourth it.

$$= 14^{4/5}$$

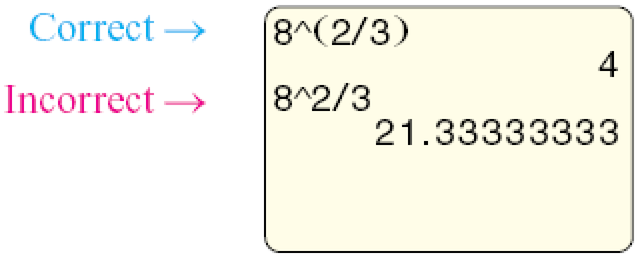
$$= \left(\sqrt[5]{14}\right)^{4}$$

Cannot be evaluated by hand.

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

TECHNOLOGY NOTE: Rational Exponents

When evaluating expressions with rational (fractional) exponents, be sure to put parentheses around the fraction. For example, most calculators will evaluate 8^(2/3) and 8^2/3 differently. The accompanying figure shows evaluation of $8^{2/3}$ input correctly, 8^(2/3), as 4 but shows evaluation of $8^{2/3}$ input incorrectly, 8^2/3, as $\frac{8^2}{3} = 21.\overline{3}$.



Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

THE EXPRESSION $a^{-m/n}$

If m and n are positive integers with $\frac{m}{n}$ in lowest terms, then

$$a^{-m/n} = \frac{1}{a^{m/n}}, \qquad a \neq 0.$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Interpreting negative rational exponents

Write each expression in radical notation and then evaluate.

a. $81^{-1/4}$

b. $64^{-2/3}$

Solution

a. $81^{-1/4} = \dfrac{1}{81^{1/4}}$

$= \dfrac{1}{\sqrt[4]{81}}$

$= \dfrac{1}{3}$

b. $64^{-2/3}$

$= \dfrac{1}{64^{2/3}}$

$= \dfrac{1}{\left(\sqrt[3]{64}\right)^2}$

$= \dfrac{1}{4^2} = \dfrac{1}{16}$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

PROPERTIES OF EXPONENTS

Let p and q be rational numbers written in lowest terms. For all real numbers a and b for which the expressions are real numbers the following properties hold.

1.	$a^p \cdot a^q = a^{p+q}$	Product rule for exponents
2.	$a^{-p} = \dfrac{1}{a^p}, \quad \dfrac{1}{a^{-p}} = a^p$	Negative exponents
3.	$\left(\dfrac{a}{b}\right)^{-p} = \left(\dfrac{b}{a}\right)^p$	Negative exponents for quotients
4.	$\dfrac{a^p}{a^q} = a^{p-q}$	Quotient rule for exponents
5.	$(a^p)^q = a^{pq}$	Power rule for exponents
6.	$(ab)^p = a^p b^p$	Power rule for products
7.	$\left(\dfrac{a}{b}\right)^p = \dfrac{a^p}{b^p}$	Power rule for quotients

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Applying properties of exponents

Write each expression using rational exponents and simplify. Write the answer with a positive exponent. Assume that all variables are positive numbers.

a. $\sqrt{x} \cdot \sqrt[4]{x}$

b. $\sqrt[4]{256x^3}$

Solution

a. $\sqrt{x} \cdot \sqrt[4]{x} = x^{1/2} \cdot x^{1/4}$

$= x^{1/2+1/4}$

$= x^{3/4}$

b. $\sqrt[4]{256x^3} = (256x^3)^{1/4}$

$= 256^{1/4}(x^3)^{1/4}$

$= 4x^{3/4}$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Applying properties of exponents-continued

c. $\dfrac{\sqrt[5]{32x}}{\sqrt[4]{x}}$

d. $\left(\dfrac{x^3}{27}\right)^{-1/3}$

Solution

a. $\dfrac{\sqrt[5]{32x}}{\sqrt[4]{x}} = \dfrac{(32x)^{1/5}}{x^{1/4}}$

$= \dfrac{32^{1/5}x^{1/5}}{x^{1/4}}$

$= 2x^{1/5-1/4}$

$= 2x^{-1/20}$

$= \dfrac{2}{x^{1/20}}$

b. $\left(\dfrac{x^3}{27}\right)^{-1/3} = \left(\dfrac{27}{x^3}\right)^{1/3}$

$= \dfrac{27^{1/3}}{(x^3)^{1/3}}$

$= \dfrac{3}{x}$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Applying properties of exponents

Write each expression with positive rational exponents and simplify, if possible.

a. $\sqrt[4]{\sqrt{x+2}}$

b. $\dfrac{y^{-1/4}}{x^{-1/5}}$

Solution

a. $\sqrt[4]{\sqrt{x+2}} = \left((x+2)^{1/2}\right)^{1/4}$

$= (x+2)^{1/8}$

b. $\dfrac{y^{-1/4}}{x^{-1/5}} = \dfrac{x^{1/5}}{y^{1/4}}$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

Practice for section 10.1

- Finding principal square roots Q 11-18
- Approximating a square root Q 35-36
- Finding nth roots Q 29-34
- Simplifying expressions Q 103-110
- Interpreting rational exponents Q 41-56,63-68
- Interpreting negative rational exponents Q 57-62
- Applying properties of exponents Q 91-102

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

10.2 Simplifying Radical Expressions

- Product Rule for Radical Expressions
- Quotient Rule for Radical Expressions

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

Consider the following example:

$$\sqrt{4}\cdot\sqrt{25} = 2\cdot 5 = 10$$

$$\sqrt{4\cdot 25} = \sqrt{100} = 10$$

PRODUCT RULE FOR RADICAL EXPRESSIONS

Let a and b be real numbers, where $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are both defined. Then

$$\sqrt[n]{a}\cdot\sqrt[n]{b} = \sqrt[n]{a\cdot b}.$$

Note: the product rule only works when the radicals have the *same index*.

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Multiplying radical expressions

Multiply each radical expression.

Solution

a. $\sqrt{36}\cdot\sqrt{4} = \sqrt{36\cdot 4} = \sqrt{144} = 12$

b. $\sqrt[3]{-8}\cdot\sqrt[3]{27} = \sqrt[3]{-8\cdot 27} = \sqrt[3]{-216} = -6$

c. $\sqrt[4]{\frac{1}{4}}\cdot\sqrt[4]{\frac{1}{16}}\cdot\sqrt[4]{\frac{1}{4}} = \sqrt[4]{\frac{1}{4}\cdot\frac{1}{16}\cdot\frac{1}{4}} = \sqrt[4]{\frac{1}{256}} = \frac{1}{4}$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE

Multiplying radical expressions containing variables

Multiply each radical expression. Assume all variables are positive.

Solution

a. $\sqrt{x^2} \cdot \sqrt{x^4} = \sqrt{x^2 \cdot x^4} = \sqrt{x^6} = x^3$

b. $\sqrt[3]{5a} \cdot \sqrt[3]{10a^2} = \sqrt[3]{5a \cdot 10a^2} = \sqrt[3]{50a^3} = a\sqrt[3]{50}$

c. $\sqrt[4]{\dfrac{3x}{y}} \cdot \sqrt[4]{\dfrac{7y}{x}} = \sqrt[4]{\dfrac{3x}{y} \cdot \dfrac{7y}{x}} = \sqrt[4]{\dfrac{21xy}{xy}} = \sqrt[4]{21}$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

SIMPLIFYING RADICALS (nth ROOTS)

STEP 1: Determine the largest perfect nth power factor of the radicand.

STEP 2: Use the product rule to factor out and simplify this perfect nth power.

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Simplifying radical expressions

Simplify each expression.

a. $\sqrt{500}$ b. $\sqrt[3]{40}$ c. $\sqrt{72}$

Solution

a. $\sqrt{500} = \sqrt{100} \cdot \sqrt{5} = 10\sqrt{5}$

b. $\sqrt[3]{40} = \sqrt[3]{8} \cdot \sqrt[3]{5} = 2\sqrt[3]{5}$

c. $\sqrt{72} = \sqrt{36} \cdot \sqrt{2} = 6\sqrt{2}$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Simplifying radical expressions

Simplify each expression. Assume that all variables are positive.

a. $\sqrt{49x^4}$ b. $\sqrt{75y^5}$ c. $\sqrt[3]{3a} \cdot \sqrt[3]{9a^2w}$

Solution

a. $\sqrt{49x^4} \quad = \sqrt{49} \cdot \sqrt{x^4} = 7x^2$

b. $\sqrt{75y^5} \quad = \sqrt{\left(25y^4\right) \cdot 3y}$

$= \sqrt{25y^4} \cdot \sqrt{3y}$

$= 5y^2\sqrt{3y}$

c. $\sqrt[3]{3a} \cdot \sqrt[3]{9a^2w} \quad = \sqrt[3]{3a \cdot 9a^2w}$

$= \sqrt[3]{\left(27a^3\right)w}$

$= \sqrt[3]{\left(27a^3\right)} \cdot \sqrt[3]{w}$

$= 3a\sqrt[3]{w}$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Multiplying radicals with different indexes

Simplify each expression.

a. $\sqrt{7} \cdot \sqrt[3]{7}$

b. $\sqrt[3]{a} \cdot \sqrt[5]{a}$

Solution

a. $\sqrt{7} \cdot \sqrt[3]{7}$ $= 7^{1/2} \cdot 7^{1/3}$

$= 7^{1/2+1/3}$

$= 7^{5/6}$

b. $\sqrt[3]{a} \cdot \sqrt[5]{a} = a^{1/3} \cdot a^{1/5}$

$= a^{1/3+1/5}$

$= a^{8/15}$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

Consider the following examples of dividing radical expressions:

$$\sqrt{\frac{4}{9}} = \sqrt{\frac{2}{3} \cdot \frac{2}{3}} = \frac{2}{3}$$

$$\sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}$$

QUOTIENT RULE FOR RADICAL EXPRESSIONS

Let a and b be real numbers, where $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are both defined and $b \neq 0$. Then

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}.$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Simplifying quotients

Simplify each radical expression. Assume that all variables are positive.

a. $\sqrt[3]{\frac{7}{27}}$

b. $\sqrt[5]{\frac{x}{32}}$

Solution

a. $\sqrt[3]{\frac{7}{27}} = \frac{\sqrt[3]{7}}{\sqrt[3]{27}}$

$= \frac{\sqrt[3]{7}}{3}$

b. $\sqrt[5]{\frac{x}{32}} = \frac{\sqrt[5]{x}}{\sqrt[5]{32}}$

$= \frac{\sqrt[5]{x}}{2}$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Simplifying quotients

Simplify each radical expression. Assume that all variables are positive.

a. $\dfrac{\sqrt{90}}{\sqrt{10}}$

b. $\dfrac{\sqrt{x^4 y}}{\sqrt{y}}$

Solution

a. $\dfrac{\sqrt{90}}{\sqrt{10}} = \sqrt{\dfrac{90}{10}}$

$= \sqrt{9}$

$= 3$

b. $\dfrac{\sqrt{x^4 y}}{\sqrt{y}} = \sqrt{\dfrac{x^4 y}{y}}$

$= \sqrt{x^4}$

$= x^2$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Simplifying radical expressions

Simplify the radical expression. Assume that all variables are positive. $\sqrt[5]{\dfrac{32x^4}{y^5}}$

Solution

$$\sqrt[5]{\frac{32x^4}{y^5}} = \frac{\sqrt[5]{32x^4}}{\sqrt[5]{y^5}}$$

$$= \frac{\sqrt[5]{32} \cdot \sqrt[5]{x^4}}{\sqrt[5]{y^5}}$$

$$= \frac{2\sqrt[5]{x^4}}{y}$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

Practice for section 10.2

- Multiplying radical expressions Q 11-22
- Multiplying radical expressions containing variables   Q 23-62 (not 33, 39,41)
- Simplifying radical expressions Q 75-80
- Multiplying radicals with different indexes Q 101-110
- Simplifying quotients Q 33,39,41
- Simplifying radical expressions Q 95-98

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

10.3 Operations on Radical Expressions

- Addition and Subtraction
- Multiplication
- Rationalizing the Denominator

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

Like radicals have the same index and the same radicand.

Like

$$3\sqrt{2} + 5\sqrt{2}$$

Unlike

$$3\sqrt{2} + 5\sqrt{3}$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Adding like radicals

If possible, add the expressions and simplify.

Solution

a. $4\sqrt{7}+8\sqrt{7} = 12\sqrt{7}$

b. $7\sqrt[3]{5}+2\sqrt[3]{5} = 9\sqrt[3]{5}$

c. $8+\sqrt{13}$ The terms cannot be added because they are not like radicals.

d. $\sqrt{6}+\sqrt{16}$ The expression contains unlike radicals and *cannot* be added.

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Finding like radicals

Write each pair of terms as like radicals, if possible.

a. $\sqrt{80}, \sqrt{125}$

b. $4\sqrt[3]{16},\ 7\sqrt[3]{54}$

Solution

a. $\sqrt{80} = \sqrt{16 \cdot 5} = 4\sqrt{5}$

$\sqrt{125} = \sqrt{25} \cdot \sqrt{5} = 5\sqrt{5}$

b. $4\sqrt[3]{16} = 4\sqrt[3]{8} \cdot \sqrt[3]{2}$

$= 4 \cdot 2\sqrt[3]{2} = 8\sqrt[3]{2}$

$7\sqrt[3]{54} = 7\sqrt[3]{27} \cdot \sqrt[3]{2}$

$= 7 \cdot 3 \cdot \sqrt[3]{2} = 21\sqrt[3]{2}$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Adding radical expressions

Add the expressions and simplify.

a. $\sqrt{20}+5\sqrt{5}$

b. $5\sqrt{2}+\sqrt{50}+\sqrt{72}$

Solution

a. $\sqrt{20}+5\sqrt{5}$

$$=\sqrt{4\cdot 5}+5\sqrt{5}$$

$$=2\sqrt{5}+5\sqrt{5}$$

$$=7\sqrt{5}$$

b. $5\sqrt{2}+\sqrt{50}+\sqrt{72}$

$$=5\sqrt{2}+\sqrt{25\cdot 2}+\sqrt{36\cdot 2}$$

$$=5\sqrt{2}+5\sqrt{2}+6\sqrt{2}$$

$$=16\sqrt{2}$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Subtracting like radicals

Simplify the expressions.

Solution

a. $8\sqrt{7} - 2\sqrt{7} \quad = 6\sqrt{7}$

b. $7\sqrt[3]{5} - 2\sqrt[3]{5} + \sqrt[3]{5} \quad = (7 - 2 + 1)\sqrt[3]{5} \quad = 6\sqrt[3]{5}$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Subtracting radical expressions

Subtract and simplify. Assume that all variables are positive.

a. $\sqrt{49x^5} - \sqrt{x^5}$

b. $\sqrt[3]{\dfrac{7y}{64}} - \dfrac{\sqrt[3]{7y}}{4}$

Solution

a. $\sqrt{49x^5} - \sqrt{x^5}$

$$= \sqrt{49x^4} \cdot \sqrt{x} - \sqrt{x^4} \cdot \sqrt{x}$$

$$= 7x^2\sqrt{x} - x^2\sqrt{x}$$

$$= 6x^2\sqrt{x}$$

b. $\sqrt[3]{\dfrac{7y}{64}} - \dfrac{\sqrt[3]{7y}}{4}$

$$= \frac{\sqrt[3]{7y}}{4} - \frac{\sqrt[3]{7y}}{4}$$

$$= 0$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Subtracting radical expressions

Subtract and simplify. Assume that all variables are positive.

a. $\dfrac{7\sqrt{2}}{5} - \dfrac{3\sqrt{2}}{3}$

b. $\sqrt[3]{343a^7b^4} - 24\sqrt[3]{27ab}$

Solution

a. $\dfrac{7\sqrt{2}}{5} - \dfrac{3\sqrt{2}}{3}$

$$= \frac{7\sqrt{2}}{5} \cdot \frac{3}{3} - \frac{3\sqrt{2}}{3} \cdot \frac{5}{5}$$

$$= \frac{21\sqrt{2}}{15} - \frac{15\sqrt{2}}{15}$$

$$= \frac{6\sqrt{2}}{15} = \frac{2\sqrt{2}}{5}$$

b. $\sqrt[3]{343a^7b^4} - \sqrt[3]{27ab}$

$$= \sqrt[3]{343a^6b^3} \cdot \sqrt[3]{ab} - \sqrt[3]{27} \cdot \sqrt[3]{ab}$$

$$= 7a^2b\sqrt[3]{ab} - 3\sqrt[3]{ab}$$

$$= (7a^2b - 3)\sqrt[3]{ab}$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Multiplying radical expressions

Multiply and simplify.

$$\left(3+\sqrt{x}\right)\left(5-\sqrt{x}\right)$$

Solution

$$\left(3+\sqrt{x}\right)\left(5-\sqrt{x}\right) = 3\cdot 5 - 3\sqrt{x} + 5\sqrt{x} - \sqrt{x}\cdot\sqrt{x}$$

$$= 15 - 3\sqrt{x} + 5\sqrt{x} - \sqrt{x^2}$$

$$= 15 + 2\sqrt{x} - x$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Rationalizing the denominator

Rationalize each denominator. Assume that all variables are positive.

a. $\dfrac{1}{\sqrt{3}}$ b. $\dfrac{7}{8\sqrt{3}}$ c. $\dfrac{ab}{\sqrt{b^5}}$

Solution

a. $\dfrac{1}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$

b. $\dfrac{7}{8\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{7\sqrt{3}}{8 \cdot 3} = \dfrac{7\sqrt{3}}{24}$

c. $\dfrac{ab}{\sqrt{b^5}} = \dfrac{ab}{b^2\sqrt{b}} = \dfrac{ab}{b^2\sqrt{b}} \cdot \dfrac{\sqrt{b}}{\sqrt{b}} = \dfrac{ab\sqrt{b}}{b^2 \cdot b} = \dfrac{a\sqrt{b}}{b^2}$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

Examples of conjugates.

TABLE 7.1

Expression	$1 + \sqrt{2}$	$\sqrt{3} - 2$	$\sqrt{x} + 7$	$\sqrt{a} - \sqrt{b}$
Conjugate	$1 - \sqrt{2}$	$\sqrt{3} + 2$	$\sqrt{x} - 7$	$\sqrt{a} + \sqrt{b}$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Using a conjugate to rationalize the denominator

Rationalize the denominator of $\dfrac{1}{1+\sqrt{3}}$.

Solution

$$\frac{1}{1+\sqrt{3}} = \frac{1}{1+\sqrt{3}} \cdot \frac{1-\sqrt{3}}{1-\sqrt{3}}$$

$$= \frac{1-\sqrt{3}}{1^2-(\sqrt{3})^2}$$

$$= \frac{1-\sqrt{3}}{1-3}$$

$$= \frac{1-\sqrt{3}}{-2}$$

$$= \frac{1}{-2} - \frac{\sqrt{3}}{-2}$$

$$= -\frac{1}{2} + \frac{\sqrt{3}}{2}$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Rationalizing the denominator

Rationalize the denominator. $$\frac{4+\sqrt{6}}{3-\sqrt{6}}$$

Solution

$$\frac{4+\sqrt{6}}{3-\sqrt{6}} = \frac{4+\sqrt{6}}{3-\sqrt{6}} \cdot \frac{3+\sqrt{6}}{3+\sqrt{6}}$$

$$= \frac{12+4\sqrt{6}+3\sqrt{6}+(\sqrt{6})^2}{9-(\sqrt{6})^2}$$

$$= \frac{18+7\sqrt{6}}{3}$$

$$= \frac{18}{3}+\frac{7\sqrt{6}}{3} \quad = 6+\frac{7\sqrt{6}}{3}$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Rationalizing the denominator having a cube root

Rationalize the denominator. $$\frac{4}{\sqrt[3]{x^2}}$$

Solution

$$\frac{4}{\sqrt[3]{x^2}} = \frac{4}{x^{2/3}}$$

$$= \frac{4}{x^{2/3}} \cdot \frac{x^{1/3}}{x^{1/3}}$$

$$= \frac{4x^{1/3}}{x^{2/3+1/3}}$$

$$= \frac{4\sqrt[3]{x}}{x}$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

Practice for section 10.3

- Adding like radicals Q 19-30
- Finding like radicals Q 9-18
- Adding radical expressions Q 29-49
- Subtracting like radicals Q 33-40
- Subtracting radical expressions Q 55-76
- Multiplying radical expressions Q 77-88
- Rationalizing the denominator Q 89-98
- Using a conjugate to rationalize the denominator Q 99-102
- Rationalizing the denominator Q 103-108
- Rationalizing the denominator having a cube root Q 113-116

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

10.6 Complex Numbers

- Basic Concepts
- Addition, Subtraction, and Multiplication
- Powers of *i*
- Complex Conjugates and Division

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

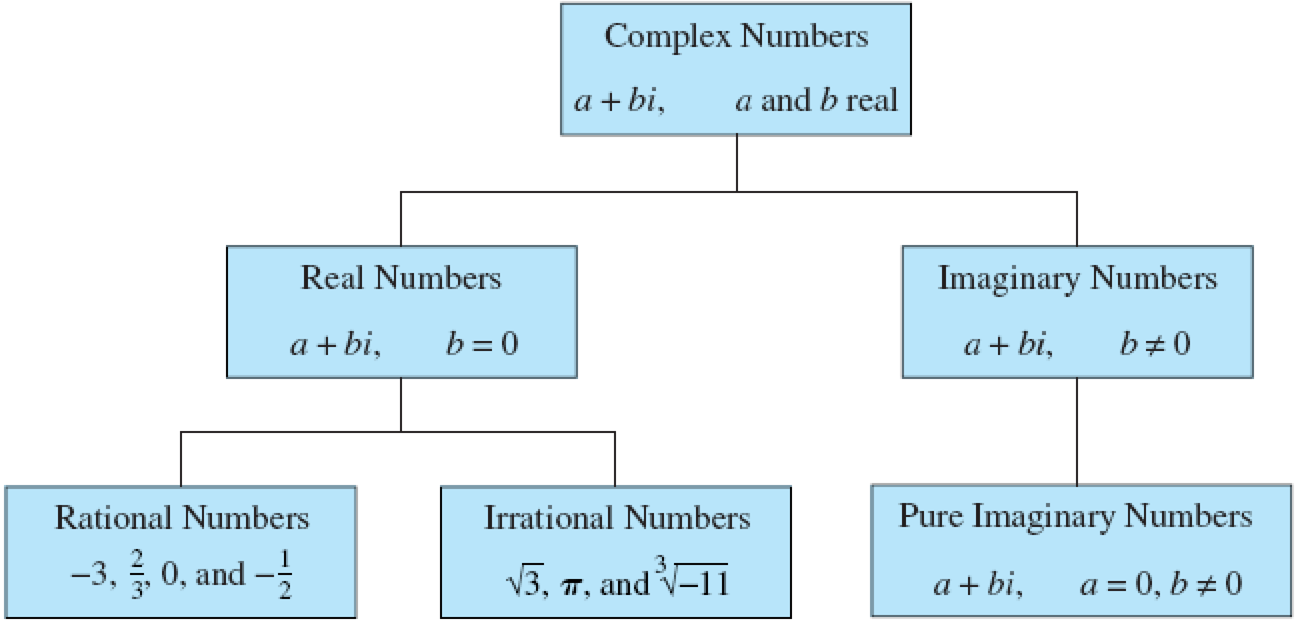
PROPERTIES OF THE IMAGINARY UNIT *i*

$$i=\sqrt{-1} \quad \text{and} \quad i^2=-1$$

THE EXPRESSION $\sqrt{-a}$

If $a>0$, then $\sqrt{-a}=i\sqrt{a}$.

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley


Complex Numbers
$a + bi$, a and b real
Real Numbers
$a + bi$, $b = 0$
Imaginary Numbers
$a + bi$, $b \neq 0$
Rational Numbers
-3, $\frac{2}{3}$, 0, and $-\frac{1}{2}$
Irrational Numbers
$\sqrt{3}$, π, and $\sqrt[3]{-11}$
Pure Imaginary Numbers
$a + bi$, $a = 0, b \neq 0$

EXAMPLE Writing the square root of a negative number

Write each square root using the imaginary *i*.

a. $\sqrt{-36}$ b. $\sqrt{-15}$ c. $\sqrt{-45}$

Solution

a. $\sqrt{-36} = i\sqrt{36} = 6i$

b. $\sqrt{-15} = i\sqrt{15}$

c. $\sqrt{-45} = i\sqrt{45} = i\sqrt{9}\sqrt{5} = 3i\sqrt{5}$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

SUM OR DIFFERENCE OF COMPLEX NUMBERS

Let $a + bi$ and $c + di$ be two complex numbers. Then

$(a + bi) + (c + di) = (a + c) + (b + d)i$ Sum

and

$(a + bi) - (c + di) = (a - c) + (b - d)i.$ Difference

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE

Adding and subtracting complex numbers

Write each sum or difference in standard form.

a. $(-8 + 2i) + (5 + 6i)$ b. $9i - (3 - 2i)$

Solution

a. $(-8 + 2i) + (5 + 6i) = (-8 + 5) + (2 + 6)i = -3 + 8i$

b. $9i - (3 - 2i) = 9i - 3 + 2i = -3 + (9 + 2)i = -3 + 11i$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Multiplying complex numbers

Write each product in standard form.

a. $(6 - 3i)(2 + 2i)$ b. $(6 + 7i)(6 - 7i)$

Solution

a. $(6 - 3i)(2 + 2i)$

$= (6)(2) + (6)(2i) - (2)(3i) - (3i)(2i)$

$= 12 + 12i - 6i - 6i^2$

$= 12 + 12i - 6i - 6(-1)$

$= 18 + 6i$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE continued

b. $(6 + 7i)(6 - 7i)$

$= (6)(6) - (6)(7i) + (6)(7i) - (7i)(7i)$

$= 36 - 42i + 42i - 49i^2$

$= 36 - 49i^2$

$= 36 - 49(-1)$

$= 85$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

POWERS OF *i*

The value of i^n can be found by dividing n (a positive integer) by 4. If the remainder is r, then

$$i^n = i^r.$$

Note that $i^0 = 1$, $i^1 = i$, $i^2 = -1$, and $i^3 = -i$.

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Calculating powers of i

Evaluate each expression

a. i^{25} b. i^{7} c. i^{44}

Solution

a. When 25 is divided by 4, the result is 6 with the remainder of 1. Thus $i^{25} = i^{1} = i$.

b. When 7 is divided by 4, the result is 1 with the remainder of 3. Thus $i^{7} = i^{3} = -i$.

c. When 44 is divided by 4, the result is 11 with the remainder of 0. Thus $i^{44} = i^{0} = 1$.

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE Dividing complex numbers

Write each quotient in standard form.

a. $\dfrac{3+2i}{5+i}$ b. $\dfrac{9}{3i}$

Solution

a. $$\frac{3+2i}{5+i}=\frac{(3+2i)(5-i)}{(5+i)(5-i)}=\frac{3(5)-3(i)+(2i)(5)-(2i)(i)}{5(5)-5(i)+5(i)-(i)(i)}$$

$$=\frac{15-3i+10i-2i^2}{25-5i+5i-i^2}=\frac{15+7i-2(-1)}{25-(-1)}$$

$$=\frac{17+7i}{26}=\frac{17}{26}+\frac{7i}{26}$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

EXAMPLE continued

b. $$\frac{9}{3i}$$

$$= \frac{9(-3i)}{(3i)(-3i)}$$

$$= \frac{-27i}{-9i^2}$$

$$= \frac{-27i}{-9(-1)}$$

$$= \frac{-27i}{9}$$

$$= -3i$$

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

Practice for section 10.6

- Writing the square root of a negative number
  Q 13-22
- Adding and subtracting complex numbers
  Q 23-30
- Multiplying complex numbers Q 31-36
- Calculating powers of *i* Q 49-56
- Dividing complex numbers Q 71-74

Copyright © 2009 Pearson Education, Inc. Publishing as Pearson Addison-Wesley

MTH 209 End of week 2

- You again have the answers to those problems not assigned
- Practice is SOOO important in this course.
- Work as much as you can with MyMathLab, the materials in the text, and on my Webpage.
- Do everything you can scrape time up for, first the hardest topics then the easiest.
- You are building a skill like typing, skiing, playing a game, solving puzzles.
- NEXT TIME: Nonlinear Equations – quadratic equations, proportion and variation problems