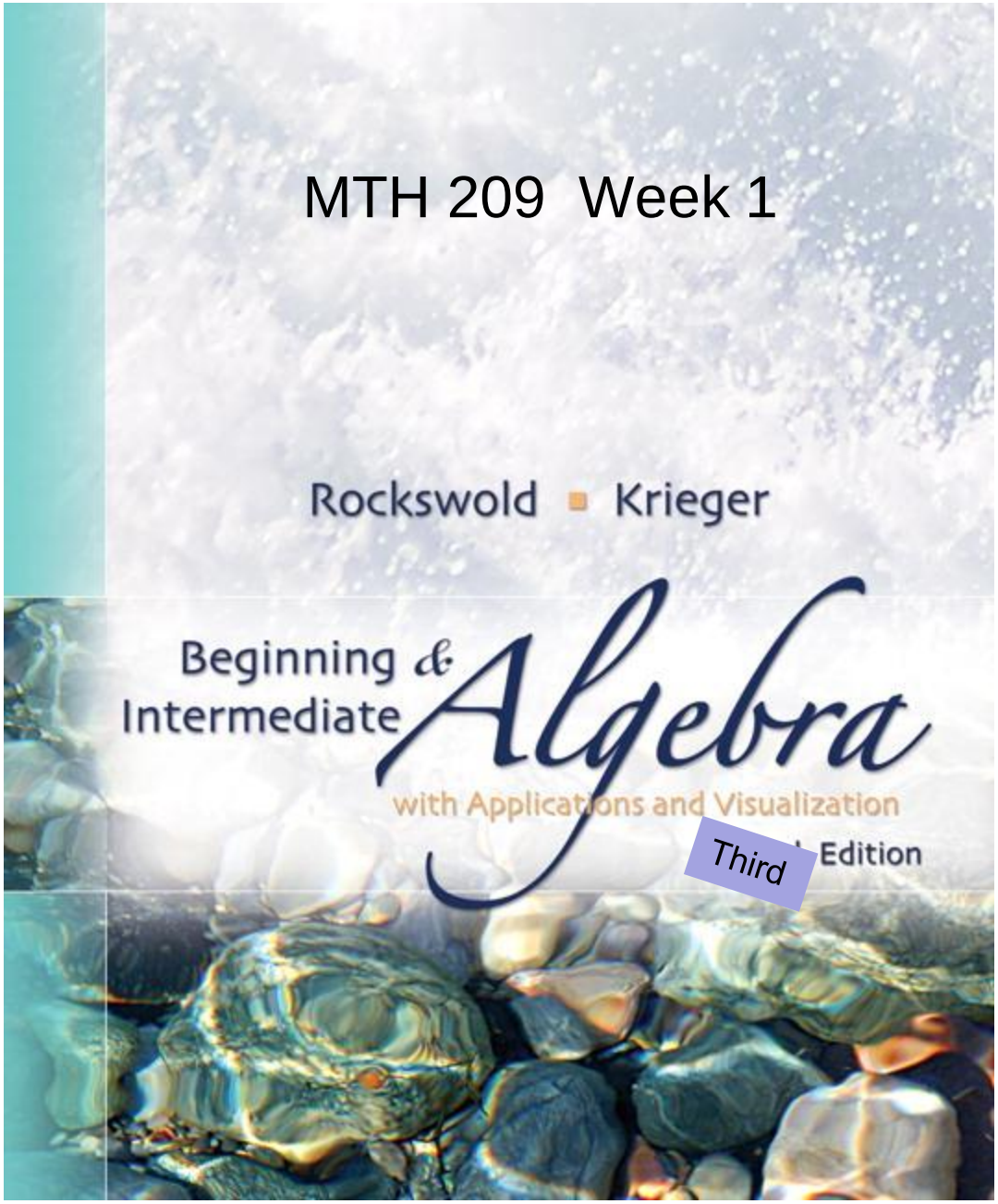
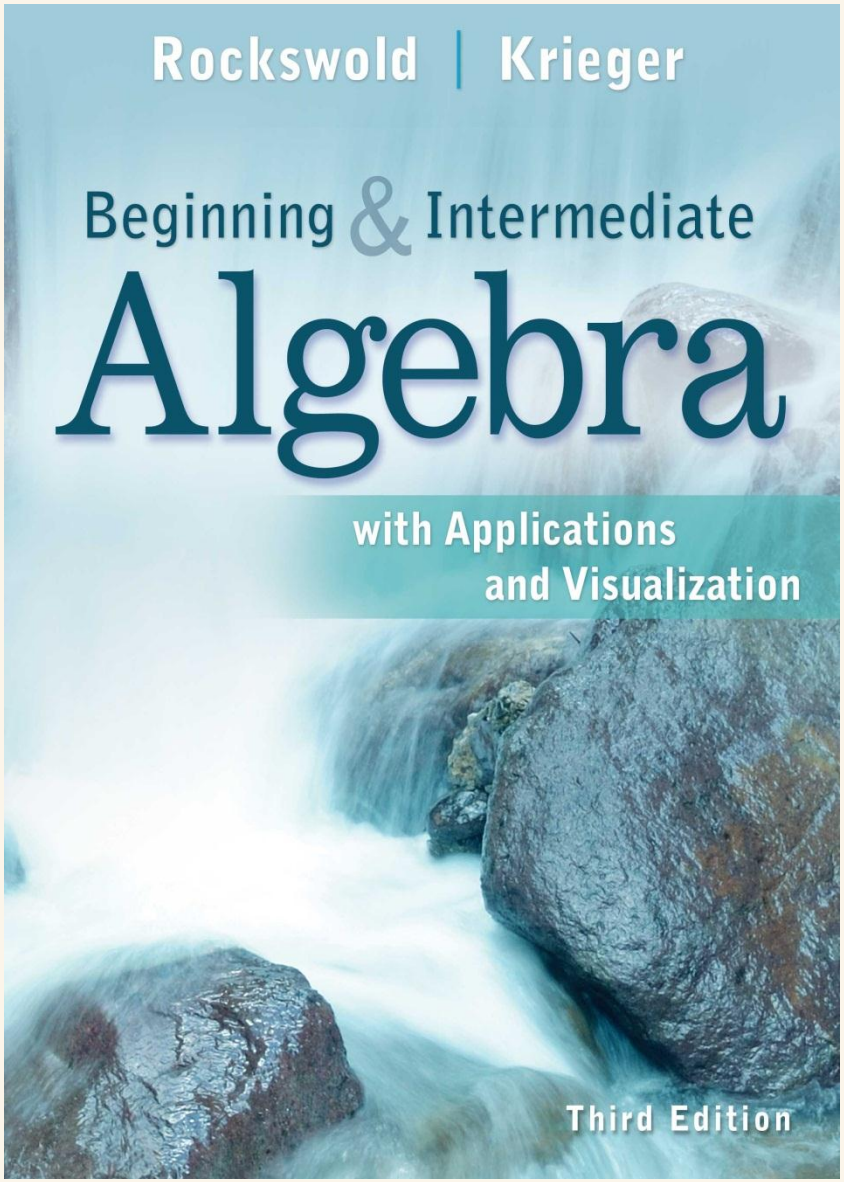
MTH 209 Week 1



Due for this week…

- Homework 1 (on MyMathLab – via the Materials Link) → The fifth night after class at 11:59pm.
- Read Chapter 6.1-6.4,
- Do the MyMathLab Self-Check for week 1.
- Learning team coordination/connections.
- Complete the Week 1 study plan after submitting week 1 homework.
- Participate in the Chat Discussions in the OLS



Section 5.2

Addition and Subtraction of Polynomials

Objectives

- Monomials and Polynomials
- Addition of Polynomials
- Subtraction of Polynomials
- Evaluating Polynomial Expressions

Monomials and Polynomials

A **monomial** is a number, a variable, or a product of numbers and variables raised to natural number powers.

Examples of monomials: $-8, \quad 7y, \quad x^3, \quad 8x^2y^9, \quad -xy^8$

The **degree of monomial** is the sum of the exponents of the variables. If the monomial has only one variable, its degree is the exponent of that variable.

The number in a monomial is called the **coefficient of the monomial.**

Example

Determine whether the expression is a polynomial. If it is, state how many terms and variables the polynomial contains and its degree.

a. $9y^2 + 7y + 4$

b. $7x^4 - 2x^3y^2 + xy - 4y^3$

c. $8x^2 - \frac{3}{x+4}$

Solution

a. The expression is a polynomial with three terms and one variable. The term with the highest degree is $9y^2$, so the polynomial has degree 2.

b. The expression is a polynomial with four terms and two variables. The term with the highest degree is $2x^3y^2$, so the polynomial has degree 5.

c. The expression is not a polynomial because it contains division by the polynomial $x + 4$.

Try Q: 21,23,27 pg 314

Example

Try Q: 29,31,33 pg 314

State whether each pair of expressions contains like terms or unlike terms. If they are like terms, then add them.

a. $9x^3, -2x^3$ b. $5mn^2, 8m^2n$

Solution

a. The terms have the same variable raised to the same power, so they are like terms and can be combined.

$$9x^3 + (-2x^3) = (9 + (-2))x^3 = 7x^3$$

b. The terms have the same variables, but these variables are not raised to the same power. They are therefore unlike terms and cannot be added.

Example

Add by combining like terms $\left(3x^2-4x+8\right)+\left(4x^2+5x-3\right)$

Solution

$$\left(3x^2-4x+8\right)+\left(4x^2+5x-3\right)=\left(3x^2-4x+8\right)+\left(4x^2+5x-3\right)$$

$$=3x^2+4x^2-4x+5x+8-3$$

$$=(3+4)x^2+(-4+5)x+(8-3)$$

$$=7x^2+x+5$$

Try Q: 37,38 pg 314

Example

Simplify. $\left(7x^2 - 3xy - 7y^2\right) + \left(-2x^2 + xy + 2y^2\right)$.

Solution

Write the polynomial in a vertical format and then add each column of like terms.

$$\begin{array}{r} 7x^2 - 3xy - 7y^2 \\ \underline{-2x^2 + xy + 2y^2} \\ 5x^2 - 2xy - 5y^2 \end{array}$$

Try Q: 41 pg 314

Subtraction of Polynomials

To subtract two polynomials, we add the first polynomial to the *opposite* of the second polynomial. To find the **opposite of a polynomial,** we negate each term.

Polynomial	Opposite
$2x - 4$	$-2x + 4$
$-x^2 - 2x + 9$	$x^2 + 2x - 9$
$6x^3 - 12$	$-6x^3 + 12$
$-3x^4 - 2x^2 - 8x + 3$	$3x^4 + 2x^2 + 8x - 3$

Example

Simplify. $\left(5w^3+3w^2-6\right)-\left(5w^3-4w^2-8\right)$.

Solution

The opposite of $\left(5w^3-4w^2-8\right)$ is $\left(-5w^3+4w^2+8\right)$

$$=\left(5w^3+3w^2-6\right)+\left(-5w^3+4w^2+8\right)$$

$$=(5-5)w^3+(3+4)w^2+(-6+8)$$

$$=0w^3+7w^2+2$$

$$=7w^2+2$$

Try Q: 57,59,61 pg 314

Example

Simplify. $\left(10x^2 - 4x + 5\right) - \left(4x^2 + 2x - 1\right)$.

Solution

$$\begin{array}{r} 10x^2 - 4x + 5 \\ -4x^2 - 2x + 1 \\ \hline 6x^2 - 6x + 6 \end{array}$$

Try Q: 69 pg 315

Example

Write a monomial that represents the total volume of three identical cubes that measure x along each edge. Find the total volume when $x = 4$ inches.

Solution

The volume of ONE cube is found by multiplying the length, width and height.

$$V = x \cdot x \cdot x$$

$$V = x^3$$

The volume of 3 cubes would be: $V := 3x^3$

Example (cont)

Write a monomial that represents the total volume of three identical cubes that measure x along each edge. Find the total volume when $x = 4$ inches.

Solution

Volume when $x = 4$ would be:

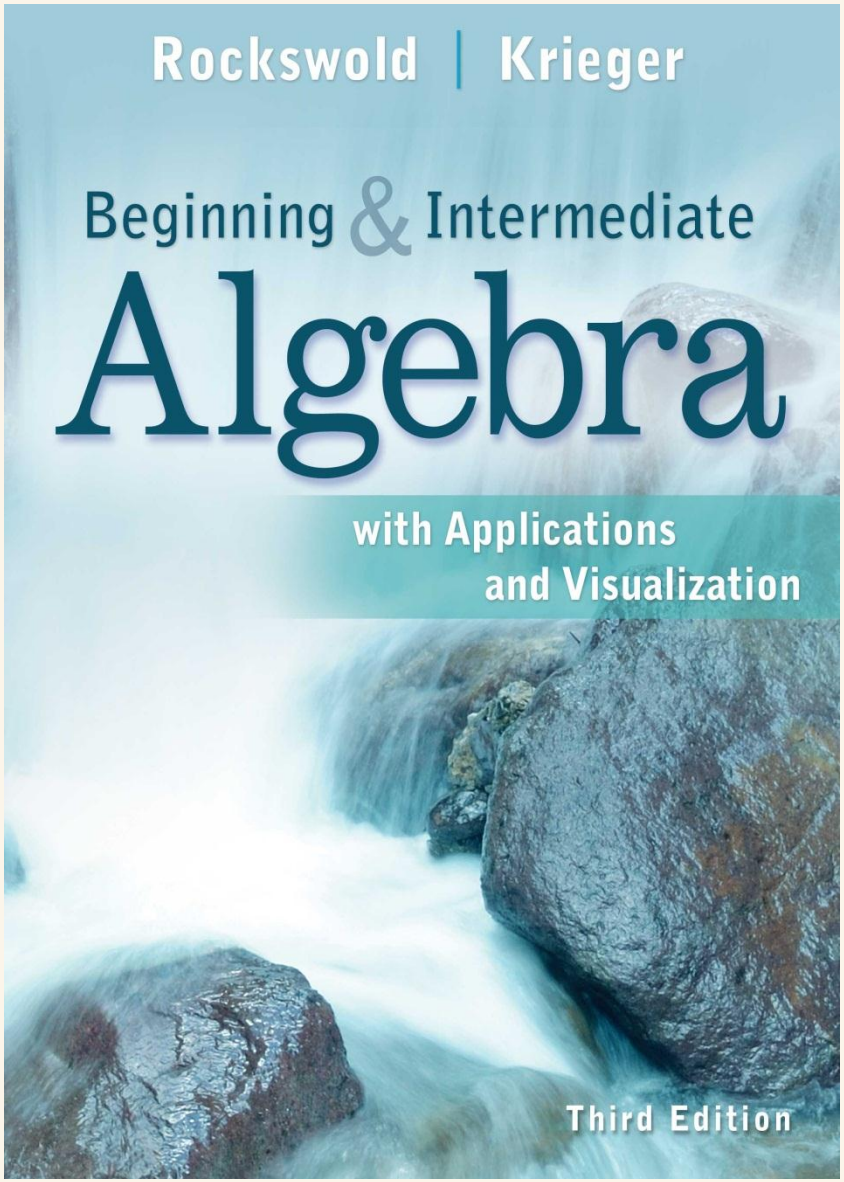
$$V = 3x^3$$

$$V = 3(4)^3$$

$$= 192$$

The volume is 192 square inches.

Try Q: 73 pg 315



Section 5.3

Multiplication of Polynomials

Objectives

- Multiplying Monomials
- Review of the Distributive Properties
- Multiplying Monomials and Polynomials
- Multiplying Polynomials

Multiplying Monomials

A monomial is a number, a variable, or a product of numbers and variables raised to natural number powers. To multiply monomials, we often use the product rule for exponents.

Example

Multiply.

a. $-6x^4 \cdot 3x^3$

b. $(6xy^3)(x^4y^2)$

Solution

a. $-6x^4 \cdot 3x^3$

$= (-6)(3)x^{4+3}$

$= -18x^7$

b. $(6xy^3)(x^4y^2)$

$= 6xx^4y^3y^2$

$= 6x^{1+4}y^{3+2}$

$= 6x^5y^5$

Try Q: 9,13 pg 322

Example

Multiply.

a. $3(6+x)$ b. $-4(x-2y)$ c. $(3x-5)(7)$

Solution

a. $3(6+x) = 3\cdot 6 + 3\cdot x$

$= 18 + 3x$

b. $-4(x-2y) = -4(x) + (-4)(-2y)$

$= -4x + 8y$

c. $(3x-5)(7) = 3x(7) - 5(7)$

$= 21x - 35$

Try Q: 15,19,21 pg 322

Example

Multiply.

a. $4xy(3x^2y - 2)$

b. $-ab(a^3 - b^3)$

Solution

a. $4xy(3x^2y - 2)$

$= 4xy \cdot 3x^2y - 4xy \cdot 2$

$= 12xx^2yy - 8xy$

$= 12x^3y^2 - 8xy$

b. $-ab(a^3 - b^3)$

$= -ab \cdot a^3 + ab \cdot b^3$

$= -a^4b + ab^4$

Try Q: 23-29 pg 322

Multiplying Polynomials

Monomials, binomials, and trinomials are examples of polynomials.

Example

Multiply. $(x+2)(x+4)$

Solution

$$(x+2)(x+4) = (x+2)(x) + (x+2)(4)$$

$$= x \cdot x + 2 \cdot x + x \cdot 4 + 2 \cdot 4$$

$$= x^2 + 2x + 4x + 8$$

$$= x^2 + 6x + 8$$

Try Q: 39 pg 323

NOTE: This process of multiplying binomials is called *FOIL*. You may use it to remind yourself to multiply the first terms (F), outside terms (O), inside terms (I), and last terms (L).

Multiply the *First terms* to obtain x^2. $(x + 1)(x + 3)$

Multiply the *Outside terms* to obtain $3x$. $(x + 1)(x + 3)$

Multiply the *Inside terms* to obtain x. $(x + 1)(x + 3)$

Multiply the *Last terms* to obtain 3. $(x + 1)(x + 3)$

The following method summarizes how to multiply two polynomials in general.

MULTIPLICATION OF POLYNOMIALS

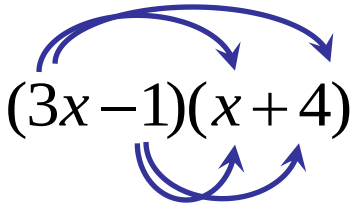
The product of two polynomials may be found by multiplying every term in the first polynomial by every term in the second polynomial.

Example

Multiply each binomial.

a. $(3x-1)(x+4)$ b $(x^2+2)(3x-1)$

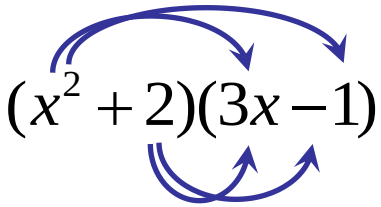
Solution

a. 

$$(3x-1)(x+4) = 3x\cdot x+3x\cdot 4-1\cdot x-1\cdot 4$$

$$=3x^2+12x-x-4$$

$$=3x^2+11x-4$$

b. 

$$(x^2+2)(3x-1) = x^2\cdot 3x+x^2\cdot(-1)+2\cdot 3x+2\cdot -1$$

$$=3x^3-x^2+6x-2$$

Try Q: 51,53,59 pg 323

Example

Multiply.

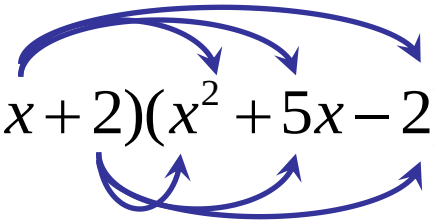
a. $4x(x^2+6x-1)$ b. $(x+2)(x^2+5x-2)$

Solution

a. 

$$4x(x^2+6x-1) = 4x\cdot x^2+4x\cdot 6x+4x\cdot -1$$

$$= 4x^3+24x^2-4x$$

b. 

$$(x+2)(x^2+5x-2) = x\cdot x^2+x\cdot 5x+x\cdot(-2)+2\cdot x^2+2\cdot 5x+2\cdot -2$$

$$= x^3+5x^2-2x+2x^2+10x-4$$

$$= x^3+7x^2+8x-4$$

Try Q: 63,67,69 pg 323

Example

Multiply. $3ab(a^2 - 3ab + 4b^2)$

Solution

$$3ab(a^2 - 3ab + 4b^2) = 3ab \cdot a^2 - 3ab \cdot 3ab + 3ab \cdot 4b^2$$

$$= 3a^3b - 9a^2b^2 + 12ab^3$$

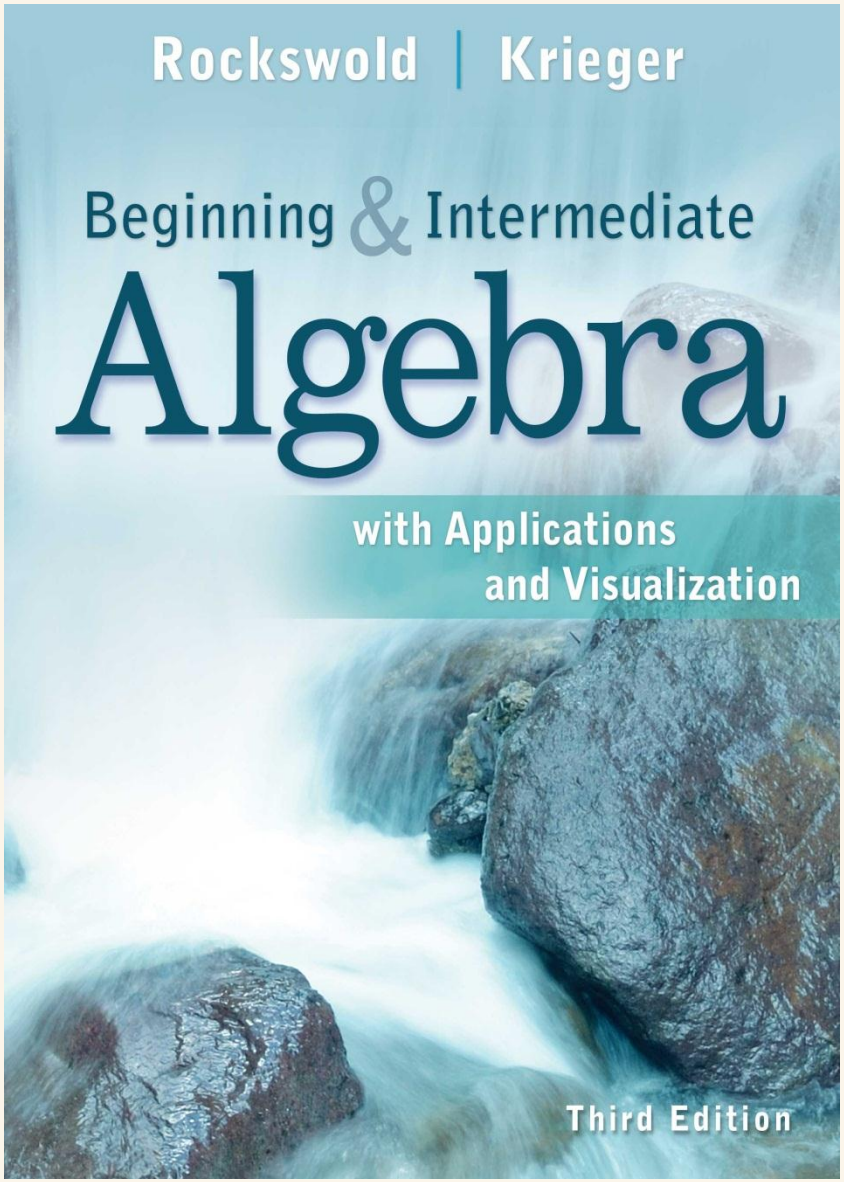
Example

Multiply vertically. $(x-1)(2x^2+x-3)$

Solution

$$\begin{array}{r} 2x^2+x-3 \\ \underline{\qquad\quad x-1} \\ -2x^2-x+3 \\ 2x^3+x^2-3x \\ \hline 2x^3-x^2-4x+3 \end{array}$$

Try Q: 71 pg 323



Section 5.4

Special Products

Objectives

- Product of a Sum and Difference
- Squaring Binomials
- Cubing Binomials

PRODUCT OF A SUM AND DIFFERENCE

For any real numbers a and b,

$$(a + b)(a - b) = a^2 - b^2.$$

Example

Multiply.

a. $(x + 4)(x - 4)$ b. $(3t + 4s)(3t - 4s)$

Solution

a. We can apply the formula for the product of a sum and difference.

$$(x + 4)(x - 4) = (x)^2 - (4)^2$$
$$= x^2 - 16$$

b. $(3t + 4s)(3t - 4s) = (3t)^2 - (4s)^2$

$$= 9t^2 - 16s^2$$

Try Q: 7,13,17 pg 329

Example

Use the product of a sum and difference to find $31 \cdot 29$.

Solution

Because $31 = 30 + 1$ and $29 = 30 - 1$, rewrite and evaluate $31 \cdot 29$ as follows.

$$\begin{aligned} 31 \cdot 29 &= (30 + 1)(30 - 1) \\ &= 30^2 - 1^2 \\ &= 900 - 1 \\ &= 899 \end{aligned}$$

Try Q: 21 pg 329

SQUARING A BINOMIAL

For any real numbers a and b,

$$(a + b)^2 = a^2 + 2ab + b^2 \quad \text{and}$$
$$(a - b)^2 = a^2 - 2ab + b^2.$$

That is, the square of a binomial equals the square of the first term, plus (or minus) twice the product of the two terms, plus the square of the last term.

Example

Multiply.

a. $(x + 7)^2$ b. $(4 - 3x)^2$

Solution

a. We can apply the formula for squaring a binomial.

$$(x + 7)^2 = (x)^2 + 2(x)(7) + (7)^2$$
$$= x^2 + 14x + 49$$

b. $(4 - 3x)^2 = (4)^2 - 2(4)(3x) + (3x)^2$

$$= 16 - 24x + 9x^2$$

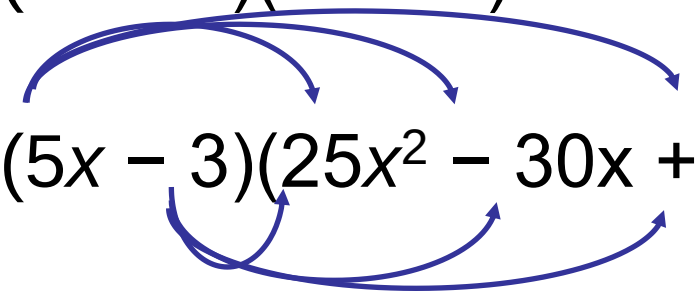
Try Q: 27,29,35,39 pg 330

Example

Multiply $(5x - 3)^3$.

Solution

$$(5x - 3)^3 = (5x - 3)(5x - 3)^2$$



$$= (5x - 3)(25x^2 - 30x + 9)$$

$$= 125x^3 - 150x^2 + 45x - 75x^2 + 90x - 27$$

$$= 125x^3 - 225x^2 + 135x - 27$$

Try Q: 47 pg 330

Example

Try Q: 75 pg 330

If a savings account pays x percent annual interest, where x is expressed as a decimal, then after 2 years a sum of money will grow by a factor of $(x + 1)^2$.

a. Multiply the expression.

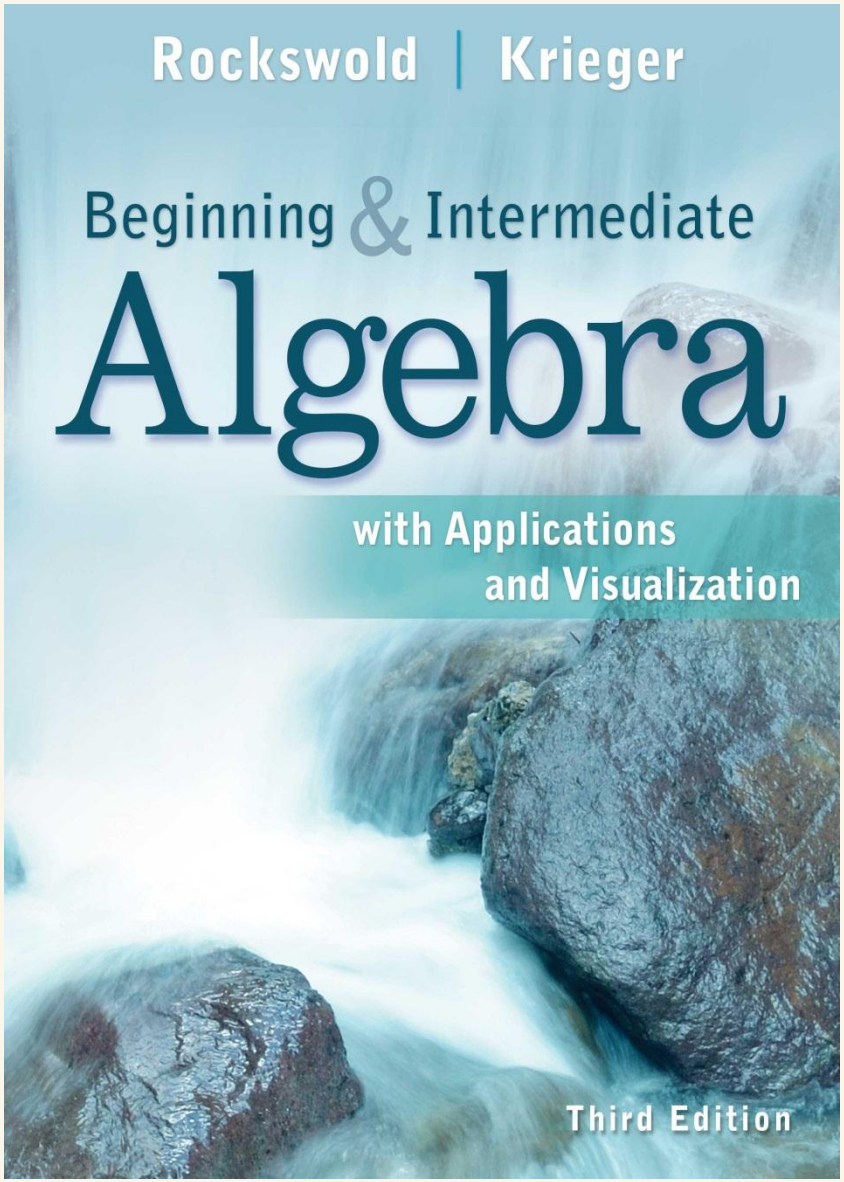
b. Evaluate the expression for $x = 0.12$ (or 12%), and interpret the result.

Solution

a. $(1 + x)^2 = 1 + 2x + x^2$

b. Let $x = 0.12$ $1 + 2(0.12) + (0.12)^2 = 1.2544$

The sum of money will increase by a factor of 1.2544. For example if \$5000 was deposited in the account, the investment would grow to \$6272 after 2 years.



Section 5.6

Dividing Polynomials

Objectives

- Division by a Monomial
- Division by a Polynomial

Example

Divide. $\dfrac{6x^5 + 18x^3}{6x^2}$

Solution

$$\frac{6x^5 + 18x^3}{6x^2} = \frac{6x^5}{6x^2} + \frac{18x^3}{6x^2} = x^3 + 3x$$

Example

Divide. $\dfrac{5a^2 - 8a + 10}{5a}$

Solution

$$\frac{5a^2 - 8a + 10}{5a} = \frac{5a^2}{5a} - \frac{8a}{5a} + \frac{10}{5a} = a - \frac{8}{5} + \frac{2}{a}$$

Try Q: 17,19,21 pg 348

Example

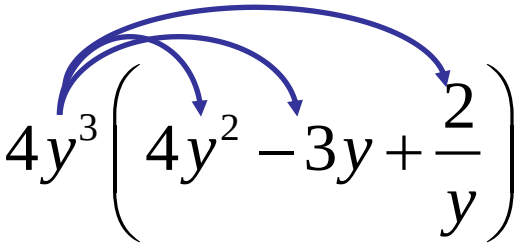
Divide the expression $\dfrac{16y^5 - 12y^4 + 8y^2}{4y^3}$ and then check the result.

Solution

$$\frac{16y^5 - 12y^4 + 8y^2}{4y^3} = \frac{16y^5}{4y^3} - \frac{12y^4}{4y^3} + \frac{8y^2}{4y^3}$$

$$= 4y^2 - 3y + \frac{2}{y}$$

Check



$$4y^3\left(4y^2 - 3y + \frac{2}{y}\right) = 4y^3 \cdot 4y^2 - 4y^3 \cdot 3y + 4y^3 \cdot \frac{2}{y}$$

$$= 16y^5 - 12y^4 + 8y^2$$

Try Q: 23 pg 348

Example

Divide and check. $\dfrac{4x^2+6x-8}{2x-1}$

Solution

$$
\begin{array}{r}
2x+4 \\
2x-1\overline{)\,4x^2+6x-8} \\
\underline{4x^2-2x} \\
8x-8 \\
\underline{8x-4} \\
-4
\end{array}
$$

The quotient is $2x + 4$ with remainder -4, which also can be written as $2x+4-\dfrac{4}{2x-1}$.

Example (cont)

$$\frac{4x^2+6x-8}{2x-1}$$

Check: (Divisor)(Quotient) + Remainder = Dividend

$(2x - 1)(2x + 4) + (-4) =$

$2x \cdot 2x + 2x \cdot 4 - 1 \cdot 2x - 1 \cdot 4 - 4$

$= 4x^2 + 8x - 2x - 4 - 4$

$= 4x^2 + 6x - 8$

It checks.

Try Q: 27 pg 349

Example

Simplify $(x^3 - 8) \div (x - 2)$.

Solution

$$
\begin{array}{r}
x^2 + 2x + 4 \\
x-2\overline{)\,x^3 + 0x^2 + 0x - 8} \\
\underline{x^3 - 2x^2} \\
2x^2 + 0x \\
\underline{2x^2 - 4x} \\
4x - 8 \\
\underline{4x - 8} \\
0
\end{array}
$$

The quotient is $x^2 + 2x + 4$.

Try Q: 37 pg 349

Example

Divide $3x^4 + 2x^3 - 11x^2 - 2x + 5$ by $x^2 - 2$.

Solution

$$
\begin{array}{r}
3x^2 + 2x - 5 \\
x^2 + 0x - 2 \overline{)\,3x^4 + 2x^3 - 11x^2 - 2x + 5} \\
\underline{3x^4 + 0 - 6x^2} \\
2x^3 - 5x^2 - 2x \\
\underline{2x^3 + 0 - 4x} \\
-5x^2 + 2x + 5 \\
\underline{-5x^2 + 0 + 10} \\
2x - 5
\end{array}
$$

The quotient is $3x^2 + 2x - 5 + \dfrac{2x-5}{x^2-2}$.

Try Q: 41 pg 349

Due for this week…

- Homework 1 (on MyMathLab – via the Materials Link) → The fifth night after class at 11:59pm.
- Read Chapter 6.1-6.4
- Do the MyMathLab Self-Check for week 1.
- Learning team planning introductions.

End of week 1

- You again have the answers to those problems not assigned
- Practice is SOOO important in this course.
- Work as much as you can with MyMathLab, the materials in the text, and on my Webpage.
- Do everything you can scrape time up for, first the hardest topics then the easiest.
- You are building a skill like typing, skiing, playing a game, solving puzzles.
- NEXT TIME: Factoring polynomials, rational expressions, radical expressions, complex numbers