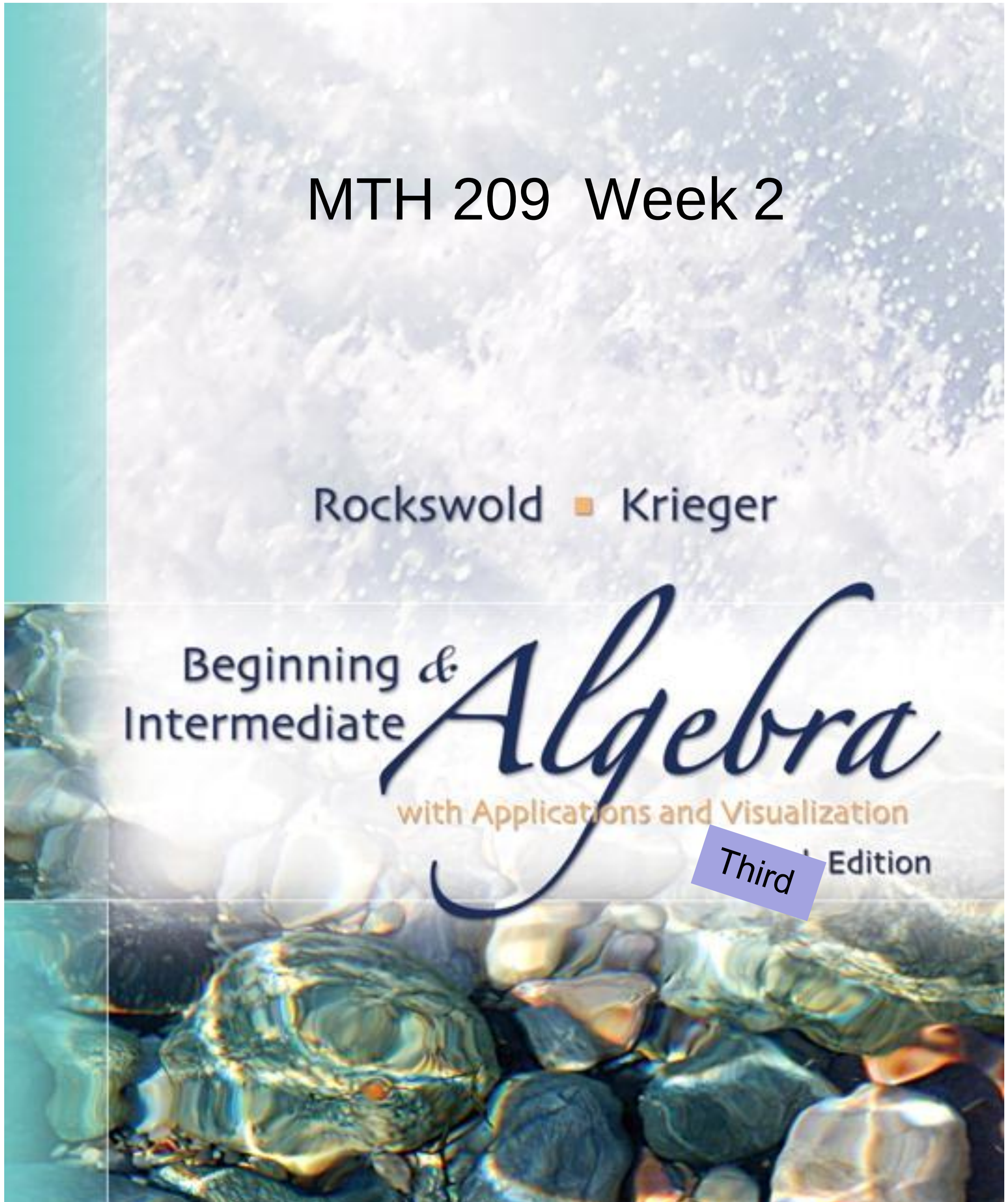
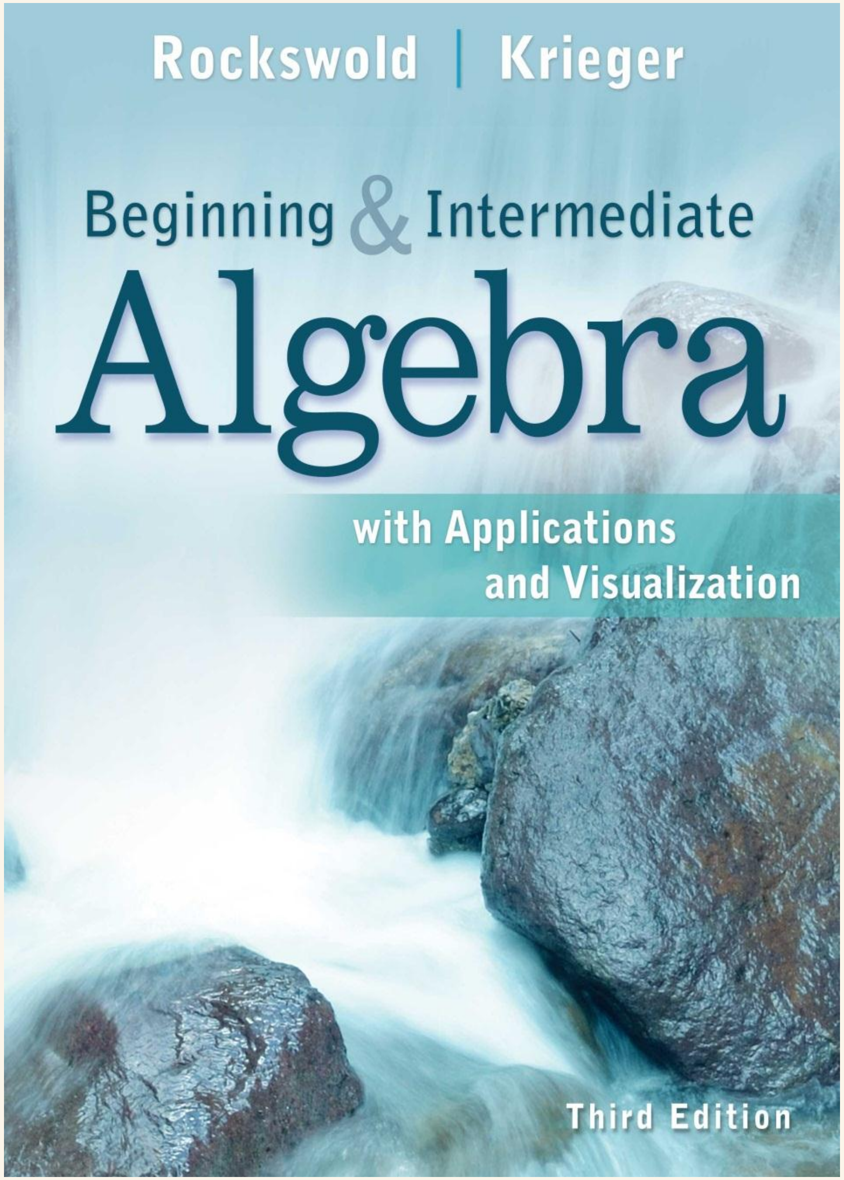
# MTH 209 Week 2



# Due for this week…

- Homework 2 (on MyMathLab – via the Materials Link) → The fifth night after class at 11:59pm.
- Read Chapter 7.1-7.4 and 7.6-7.7 and 10.1-10.3
- Do the MyMathLab Self-Check for week 2.
- Learning team hardest problem assignment.
- Complete the Week 2 study plan after submitting week 2 homework.
- Participate in the Chat Discussions in the OLS




# Section 6.1

## Introduction to Factoring

# Objectives

- Common Factors
- Factoring by Grouping

# Common Factors

When factoring a polynomial, we first look for factors that are common to each term.

By applying a distributive property we can often write a polynomial as a product.

For example: $8x^2 = 2x \cdot 4x$ and $6x = 2x \cdot 3$

And by the distributive property,
$8x^2 + 6x = 2x(4x + 3)$

# Example

Factor.

a. $6x^2 + 7x$ b. $15x^3 - 5x^2$ c. $8w^3 - 2w^2 + 4w$ d. $6x^2y^3 + x^2y^2$

**Solution**

a. $6x^2 + 7x$

$6x^2 = 6x \cdot x$

$7x = 7 \cdot x$

$x(6x + 7)$

b. $15x^3 - 5x^2$

$15x^3 = 5 \cdot 3 \cdot x^2 \cdot x$

$-5x^2 = -5 \cdot x^2$

$5x^2(3x - 1)$

# Example (cont)

Factor.

a. $6x^2 + 7x$ b. $15x^3 - 5x^2$ c. $8w^3 - 2w^2 + 4w$ d. $6x^2y^3 + x^2y^2$

**Solution**

c. $8w^3 - 2w^2 + 4w$

$8w^3 = 4 \cdot 2 \cdot w^2 \cdot w$

$-2w^2 = -2 \cdot w \cdot w$

$4w = 2 \cdot 2 \cdot w$

$= 2w(4w^2 - w + 2)$

d. $6x^2y^3 + x^2y^2$

$6x^2y^3 = 6x^2y^2$

$x^2y^2 = x^2y^2$

$x^2y^2(6y + 1)$

Try Q:9,11 page 366

# Example

Find the greatest common factor for the expression. Then factor the expression. $18x^2 + 3x$

**Solution**

$18x^2 = 2 \cdot 3 \cdot 3 \cdot x \cdot x$

$3x = 3 \cdot x$

The greatest common factor is $3x$.

$18x^2 + 3x = 3x(6x + 1)$

## Example

Find the greatest common factor for the expression. Then factor the expression. $5y^4z^3 + 20y^3z^2$

**Solution**

$$5y^4z^3 = 5 \cdot y \cdot y \cdot y \cdot y \cdot z \cdot z \cdot z$$

$$20y^3z^2 = 2 \cdot 2 \cdot 5 \cdot y \cdot y \cdot y \cdot z \cdot z$$

The greatest common factor is $5y^3z^2$.

$$5y^4z^3 + 20y^3z^2 = 5y^3z^2(yz + 4)$$

Try Q:23,25,27 page 367

# Factoring by Grouping

Factoring by grouping is a technique that makes use of the associative and distributive properties.

# Example

Factor.

a. $3x(x + 1) + 4(x + 1)$ b. $3x^2(2x - 1) - x(2x - 1)$

**Solution**

a. Both terms in the expression contain the binomial $x + 1$. Use the distributive property to factor.

$$3x(x+1)+4(x+1) = (3x+4)(x+1)$$

b. $3x^2(2x-1)-x(2x-1)$

$$3x^2(2x-1)-x(2x-1) = (3x^2-x)(2x-1)$$

$$= x(3x-1)(2x-1)$$

Try Q:41, 45 page 367

# Example

Factor the polynomial. $x^3 - 4x^2 + 3x - 12$

## Solution

$$x^3 - 4x^2 + 3x - 12 = (x^3 - 4x^2) + (3x - 12)$$

$$= (x^3 - 4x^2) + (3x - 12)$$

$$= x^2(x - 4) + 3(x - 4)$$

$$= (x^2 + 3)(x - 4)$$

Try Q:47,65 page 367

# Example

Factor the polynomial. $15x^3 - 10x^2 - 3x + 2$

**Solution**

$$15x^3 - 10x^2 - 3x + 2 = (15x^3 - 10x^2) + (-3x + 2)$$

$$= 5x^2(3x - 2) - 1(3x - 2)$$

$$= (5x^2 - 1)(3x - 2)$$

Try Q:55,61 page 367

# Example

Completely factor the polynomial. $30x^3 - 20x^2 - 6x + 4$

**Solution**

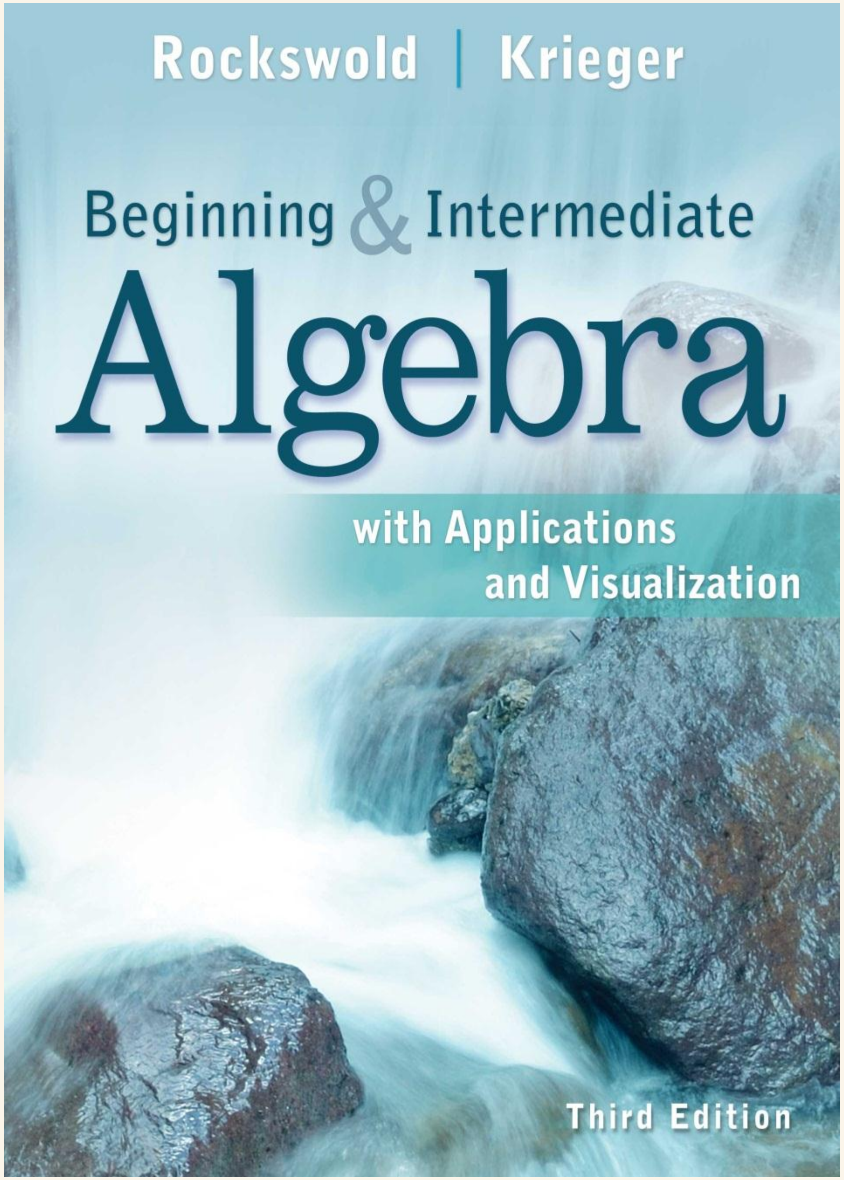
$$30x^3 - 20x^2 - 6x + 4 = 2(15x^3 - 10x^2 - 3x + 2)$$

$$= 2[(15x^3 - 10x^2) + (-3x + 2)]$$

$$= 2[5x^2(3x - 2) - 1(3x - 2)]$$

$$= 2(5x^2 - 1)(3x - 2)$$

Try Q:67,71 page 367




# Section 6.2

## Factoring Trinomials I

## $x^2 + bx + c$

# Objectives

- Review of the FOIL Method
- Factoring Trinomials Having a Leading Coefficient of 1

## FACTORING $x^2 + bx + c$

To factor the trinomial $x^2 + bx + c$, find numbers $m$ and $n$ that satisfy

$$m \cdot n = c \quad \text{and} \quad m + n = b.$$

Then $x^2 + bx + c = (x + m)(x + n)$.

# Example

Factor each trinomial.

a. $x^2 + 11x + 18$ b. $x^2 + 10x + 24$

**Solution**

a. $x^2 + 11x + 18$

Factors of 18 whose sum is 11.

| Factors | Sum |
|---|---|
| 1, 18 | 19 |
| **2, 9** | **11** |
| 3, 6 | 9 |

$(x+2)(x+9)$

b. $x^2 + 10x + 24$

Factors of 24 whose sum is 10.

| Factors | Sum |
|---|---|
| 1, 24 | 25 |
| 2, 12 | 14 |
| 3, 8 | 11 |
| **4, 6** | **10** |

$(x+4)(x+6)$

# Example

Try Q:19,21,23 page 374

Factor each trinomial.

a. $x^2 - 9x + 14$

b. $x^2 - 19x + 48$

**Solution**

a. $x^2 - 9x + 14$

Factors of 14 whose sum is −9.

| Factors | Sum |
|---|---|
| −1, −14 | −15 |
| **−2, −7** | **−9** |

$(x-2)(x-7)$

b. $x^2 - 19x + 48$

Factors of 48 whose sum is −19.

| Factors | Sum |
|---|---|
| −1, −48 | −49 |
| −2, −24 | −26 |
| **−3, −16** | **−19** |
| −4, −12 | −16 |
| −6, −8 | −14 |

$(x-3)(x-16)$

# Example

Try Q:27,29,31 page 374

Factor each trinomial.

a. $x^2 + 2x - 15$

b. $x^2 - 6x - 16$

**Solution**

a. $x^2 + 2x - 15$

Factors of −15 whose sum is 2.

| Factors | Sum |
|---|---|
| 1, −15 | −14 |
| − 1, 15 | +14 |
| 3, −5 | −2 |
| **−3, 5** | **2** |

$(x-3)(x+5)$

b. $x^2 - 6x - 16$

Factors of −16 whose sum is −6.

| Factors | Sum |
|---|---|
| −1, 16 | 15 |
| 1, −16 | −15 |
| −2, 8 | 6 |
| **2, −8** | **−6** |
| −4, 4 | 0 |

$(x+2)(x-8)$

## Example

Try Q:23,25,27 page 374

Factor the trinomial. $x^2 + 6x - 8$

**Solution**

$x^2 + 6x - 8$

Factors of −8 whose sum is 6.

| **Factors** | **Sum** |
|---|---|
| 1, −8 | −7 |
| −1, 8 | 7 |
| 2, −4 | −2 |
| −2, 4 | 2 |

The table reveals that no such factor pair exists. Therefore, the trinomial is prime.

# Example

Try Q:37,53  page 374

Factor each trinomial.

a. $4x^2+28x+48$

b. $7x^2+21x-70$

**Solution**

a. $4x^2+28x+48$

Factor out common factor of 4.

$4(x^2+7x+12)$

| Factors (12) | Sum (7) |
|---|---|
| 1, 12 | 13 |
| 2, 6 | 8 |
| **3, 4** | **7** |

$4(x+3)(x+4)$

b. $7x^2+21x-70$

Factor out common factor of 7.

$7(x^2+3x-10)$

| Factors (–10) | Sum (3) |
|---|---|
| 1, –10 | –9 |
| 2, –5 | –3 |
| **–2, 5** | **3** |

$7(x-2)(x+5)$

# Example

Find one possibility for the dimensions of a rectangle that has an area of $x^2 + 12x + 35$.

**Solution**

The area of a rectangle equals length times width. If we can factor $x^2 + 12x + 35$, then the factors can represents its length and width.

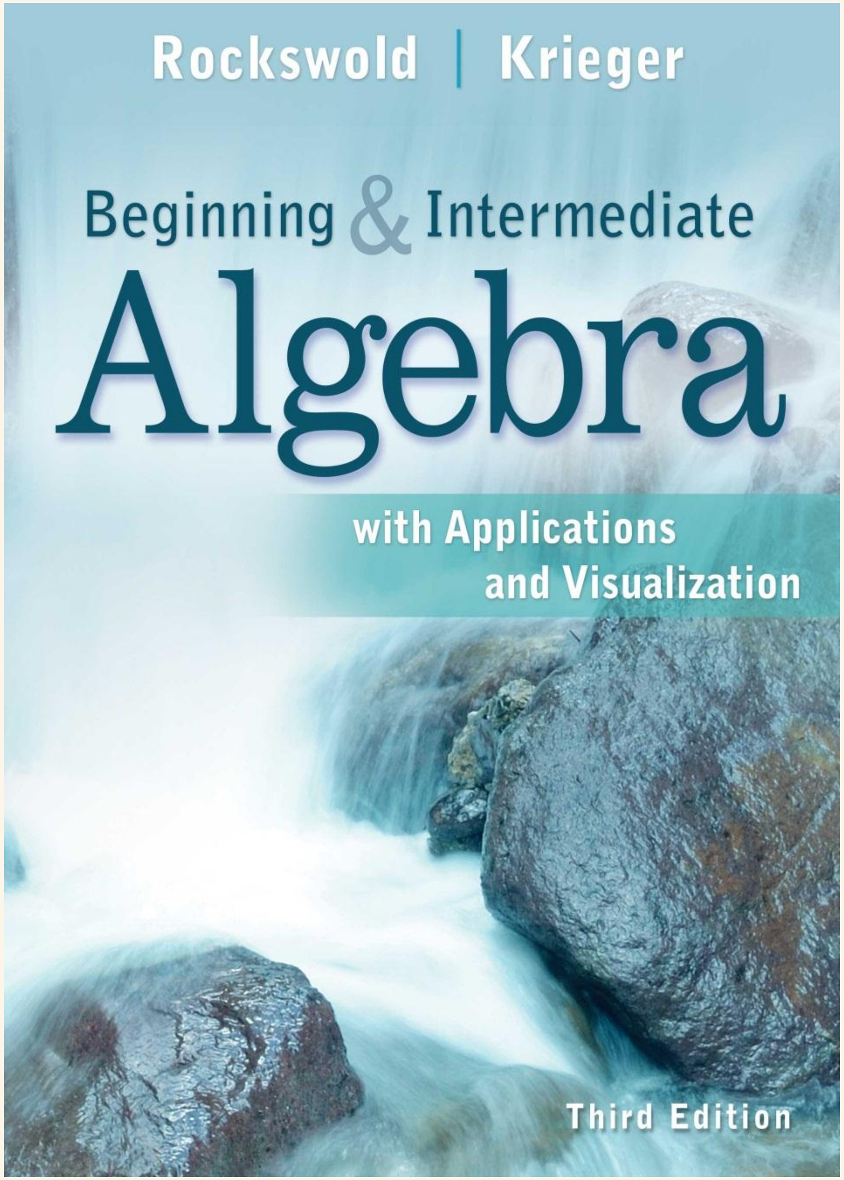
Because

$$x^2 + 12x + 35 = (x + 5)(x + 7),$$

one possibility for the rectangle's dimensions is width $x + 5$ and length $x + 7$.

Area = $x^2 + 12x + 35$

Try Q:87 page 375




# Section 6.3

## Factoring Trinomials II $ax^2 + bx + c$

# Objectives

- Factoring Trinomials by Grouping
- Factoring with FOIL in Reverse

## FACTORING $ax^2 + bx + c$ BY GROUPING

To factor $ax^2 + bx + c$ perform the following steps. (Assume that $a$, $b$, and $c$ have no factor in common.)

1. Find numbers $m$ and $n$ such that $mn = ac$ and $m + n = b$. (This step may require trial and error.)
2. Write the trinomial as $ax^2 + mx + nx + c$.
3. Use grouping to factor this expression as two binomials.

# Example

Factor each trinomial.

a. $2x^2+13x+15$

b. $12y^2-5y-3$

**Solution**

a. $2x^2+13x+15$

Multiply (2)(15) = 30
Factors of 30 whose sum is 13 = 10 and 3

$$=2x^2+10x+3x+15$$
$$=(2x^2+10x)+(3x+15)$$
$$=2x(x+5)+3(x+5)$$
$$=(2x+3)(x+5)$$

b. $12y^2-5y-3$

Multiply (12)(–3) = –36
Factors of –36 whose sum is –5 = –9 and 4

$$=12y^2-9y+4y-3$$
$$=(12y^2-9y)+(4y-3)$$
$$=3y(4y-3)+1(4y-3)$$
$$=(3y+1)(4y-3)$$

# Example

Try Q:15,25,37    page 382

Factor the trinomial. $3x^2 + 9x + 4$

**Solution**

a. We need to find integers *m* and *n* such that *mn* = (3)(4) = 12 and *m* + *n* = 9. Because the middle term is positive, we consider only positive factors of 12.

| Factors | 1, 12 | 2, 6 | 3, 4 |
|---|---|---|---|
| Sum | 13 | 8 | 7 |

There are no factors whose sum is 9, the coefficient of the middle term. The trinomial is prime.

# Factoring with FOIL in Reverse

$$3x^2 + 7x + 2 \stackrel{?}{=} (\_\_\_\_ + \_\_\_\_)(\_\_\_\_ + \_\_\_\_)$$

$$3x^2 + 7x + 2 \stackrel{?}{=} (\underline{3x} + \_\_\_\_)(\underline{x} + \_\_\_\_).$$

$$(3x + 2)(x + 1) = 3x^2 + 5x + 2$$

$$2x + 3x = 5x$$

$5x$ ⟵ Middle term is *not* $7x$.

$$(3x + 1)(x + 2) = 3x^2 + 7x + 2$$

$$1x + 6x = 7x$$

$7x$ ⟵ Middle term checks.

# Example

Factor the trinomial. $2x^2 + 7x + 6$

**Solution**

$$2x^2 + 7x + 6$$

$$2x^2 + 7x + 6 = (2x + \_\_\_)(x + \_\_\_)$$

The factors of the last term are either 1 and 6 or 2 and 3.

Try a set of factors. Try 1 and 6.

$$(2x + 1)(x + 6) = 2x^2 + 13x + 6$$

$$\begin{array}{r} x \\ \underline{12x} \\ 13x \end{array}$$

Middle term is 13*x* not 7*x*.

# Example (cont)

Try Q:19,33,69 page 382

The factors of the last term are either 1 and 6 or 2 and 3.

Try a set of factors.

Try 2 and 3 the factors of the last term.

$$(2x+2)(x+3) = 2x^2 + 8x + 6$$

$$\begin{array}{r} 2x \\ \underline{6x} \\ 8x \end{array}$$

Middle term is 8*x* not 7*x*.

Try another set of factors 3 and 2.

$$(2x+3)(x+2) = 2x^2 + 7x + 6$$

$$\begin{array}{r} 3x \\ \underline{4x} \\ 7x \end{array}$$

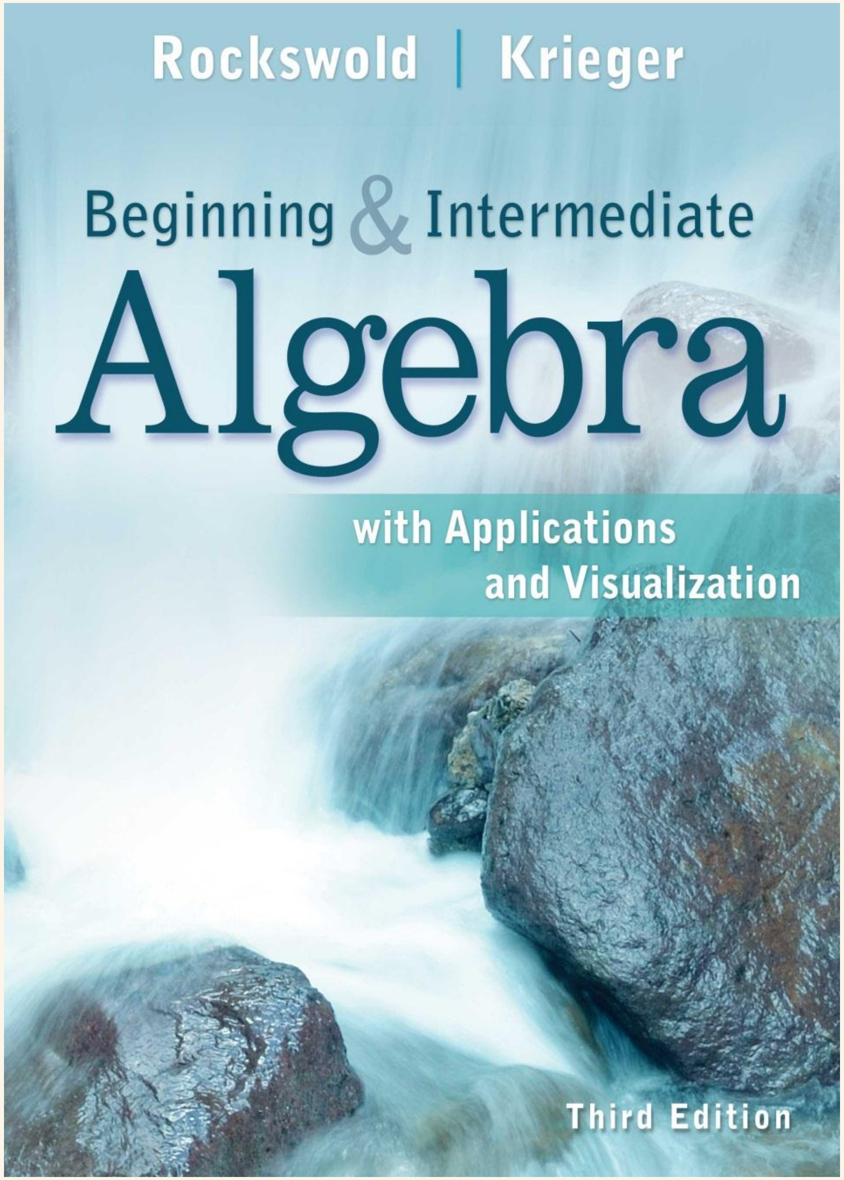
Middle term is correct.

MAKING CONNECTIONS

## The Signs in the Binomial Factors

Let $a$, $b$, and $c$ represent positive integers. If a trinomial of the form $ax^2 + bx + c$ can be factored, the signs in the binomial factors can be summarized as follows.

| *Form of the Trinomial* | *Signs in the Binomial Factors* |
| --- | --- |
| $ax^2 + bx + c$ | $(\ +\ )(\ +\ )$ |
| $ax^2 - bx + c$ | $(\ -\ )(\ -\ )$ |
| $ax^2 + bx - c$ | $(\ -\ )(\ +\ )$ |
| $ax^2 - bx - c$ | $(\ -\ )(\ +\ )$ |




# Section 6.4

## Special Types of Factoring

# Objectives

- Difference of Two Squares
- Perfect Square Trinomials
- Sum and Difference of Two Cubes

## DIFFERENCE OF TWO SQUARES

For any real numbers $a$ and $b$,

$$a^2 - b^2 = (a - b)(a + b).$$

# Example

Try Q:17,19,25,29 page 389

Factor each difference of two squares.

a. $9x^2 - 16$ b. $5x^2 + 8y^2$ c. $25x^2 - 16y^2$

**Solution**

a. $9x^2 - 16 = (3x)^2 - (4)^2 = (3x - 4)(3x + 4)$

b. Because $5x^2 + 8y^2$ is the sum of two squares, it *cannot* be factored.

c. $25x^2 - 16y^2 = (5x - 4y)(5x + 4y)$

# PERFECT SQUARE TRINOMIALS

For any real numbers $a$ and $b$,

$$a^2 + 2ab + b^2 = (a + b)^2 \text{ and}$$

$$a^2 - 2ab + b^2 = (a - b)^2.$$

# Example

If possible, factor each trinomial as a perfect square.

a. $x^2 + 8x + 16$

b. $4x^2 - 12x + 9$

**Solution**

a. $x^2 + 8x + 16$

Start by writing as $x^2 + 8x + 4^2$

Check the middle term

$2(x)(4) = 8x$, the middle term checks

$$x^2 + 8x + 16 = (x + 4)^2$$

# Example (cont)

Try Q:37,41,43,51 page 390

If possible, factor each trinomial as a perfect square.

a. $x^2 + 8x + 16$

b. $4x^2 - 12x + 9$

**Solution**

b. $4x^2 - 12x + 9$

Start by writing as $(2x)^2 - 12x + 3^2$

Check the middle term

$2(2x)(3) = 12x$, the middle term checks

$$4x^2 - 12x + 9 = (2x - 3)^2$$

# Example

If possible, factor the trinomial as a perfect square.

$$4w^2 - 14w + 49$$

**Solution**

Start by writing as $(2w)^2 - 14w + 7^2$

Check the middle term

$2(2w)(7) = 28x$, the middle term does not check

The expression cannot be factored as a perfect square trinomial.

## SUM AND DIFFERENCE OF TWO CUBES

For any real numbers $a$ and $b$,

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2) \quad \text{and}$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

# Example

Factor each polynomial.

a. $n^3 + 27$

b. $8x^3 - 125y^3$

**Solution**

a. $n^3 + 27$

Because $n^3 = (n)^3$ and $27 = 3^3$, we let $a = n$, $b = 3$, and factor.

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2) \text{ gives}$$

$$n^3 + 3^3 = (n + 3)(n^2 - n \cdot 3 + 3^2)$$

$$= (n + 3)(n^2 - 3n + 9)$$

# Example (cont)

Try Q:57,65 page 390

Factor each polynomial.

a. $n^3 + 27$

b. $8x^3 - 125y^3$

**Solution**

b. $8x^3 - 125y^3$

$8x^3 = (2x)^3$ and $125y^3 = (5y)^3$, so

$8x^3 - 125y^3 = (2x)^3 - (5y)^3$

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ gives

$$(2x)^3 - (5y)^3 = (2x - 5y)(4x^2 + 10xy + 25y^2)$$

# Example

Try Q:31,47,67 page 390

Factor the polynomial $8s^3 - 32st^2$.

**Solution**

Factor out the common factor of $8s$.

$$8s^3 - 32st^2 = 8s(s^2 - 4t^2)$$

$$= 8s(s - 2t)(s + 2t)$$

# End of week 2

- You again have the answers to those problems not assigned
- Practice is SOOO important in this course.
- Work as much as you can with MyMathLab, the materials in the text, and on my Webpage.
- Do everything you can scrape time up for, first the hardest topics then the easiest.
- You are building a skill like typing, skiing, playing a game, solving puzzles.
- NEXT TIME: Factoring polynomials, rational expressions, radical expressions, complex numbers