University of Phoenix MTH 209 Algebra II

The FUN continues!

Chapter 4

- What we are doing now is using all we have used so far and add a few complications like the concept of a root (square root, cube root, etc.).
- The big word for MTH 209 is a polynomial (poly = parrot)
- (no… poly = many, nomial = number)

Section 4.1

- Remember the Product and Quotient Rules

$a^m * a^n = a^{m+n}$

And the zero exponent

and $a^0 = 1$

Taking what they throw at ya: Ex 1 page 256

- a) $2^3 \cdot 2^2 = 2^{3+2} = 2^5 = 32$ (calculator!)

- b) $x^2 \cdot x^4 \cdot x = x^2 \cdot x^4 \cdot x^1 = x^7$

- c) $2y^3 \cdot 4y^8 = (2)(4)y^3y^8 = 8y^{11}$

- d) $-4a^2b^3(-3a^5b^9) = (-4)(-3)\ a^2a^5b^3b^9 = 12a^7b^{12}$

Ex. 7-18

Zero Exponent Example 2 page 257

a) $5^0 = 1$

b) $(3xy)^0 = 1$

c) $b^0 \cdot b^9 = 1 \cdot b^9$

c) $2^0 + 3^0 = 1 + 1 = 2$

Ex. 19-28

Section 4.1

- Remember the Quotient Rules

If $m \geq n$ then $\dfrac{a^m}{a^n} = a^{m-n}$

If n>m, then $\dfrac{a^m}{a^n} = \dfrac{1}{a^{n-m}}$

More thrown at ya: Ex 3 page 258

- a) $x^7/x^4 = x^{7-4} = x^3$
- b) $w^5/w^3 = w^{5-3} = w^2$
- c) $$\frac{2x^9}{-4x^3} = -\frac{2}{4}\cdot\frac{x^9}{x^3} = -\frac{1}{2}\cdot x^{9-3} = -\frac{1}{2}\cdot x^6 = -\frac{x^6}{2}$$
- d) $$\frac{6a^{12}b^6}{-3a^9b^6} = \frac{6a^{12}b^6}{-3a^9b^6} = -2a^{12-9}b^{6-6} = -2a^3$$

Ex. 29-40

What about raising an exponent to and exponent?

- Yes, we have to try to break it if we can!
- What ABOUT raising an exponent to a power?
- You MULTIPLY the exponents!

Power of a Power

- $(w^4)^3 = w^4 \, w^4 \, w^4 = w^{12}$
- Which is the same as $w^{4*3} = w^{12}$
- Definition time: *The Power Rule*

$$(a^m)^n = a^{mn}$$

Example 4 page 259

- a) $(2^3)^8 = 2^{3\cdot 8} = 2^{24}$

- b) $(x^2)^5 = x^{2\cdot 5} = x^{10}$

- c) $3x^8(x^3)^6 = 3x^8(x^{3\cdot 6}) = 3x^8(x^{18}) = 3x^{8+18} = 3x^{26}$

- d) $$\frac{-6(b^4)^3}{3b^2} = \frac{-6b^{12}}{3b^2} = -2b^{10}$$

Ex. 41-50

Now, to the power of a product. What if there is more than just x inside the ()’s?

- For example: $(2x)^3 = 2x2x2x = 2*2*2*x*x*x = 2^3x^3$ (then do the math and get) $8x^3$

So the *Power of a Product Rule*

$(ab)^n = a^nb^n$

Example 5 page 259

- a) $(-2x)^3=(-2)^3x^3=-8x^3$

- b) $(-3a^2)^4= (-3)^4 (a^2)^4 = 81a^8$

- c) $(5x^3y^2)^3 = 5^3(x^3)(y^2)^3=124x^9y^6$

Ex. 51-58

The power of a quotient

- It is JUST what what you'd expect!
- Raise the top and the bottom to the power by themselves, then work with it!

For example $$\left(\frac{x}{5}\right)^3 = \frac{x}{5}\cdot\frac{x}{5}\cdot\frac{x}{5} = \frac{x\cdot x\cdot x}{5\cdot 5\cdot 5} = \frac{x^3}{5^3}$$

The Power of a Quotient

- Where b does NOT equal 0

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Example 6

- a) $$\left(\frac{y}{4}\right)^3 = \frac{y^3}{4^3} = \frac{y^3}{64}$$

- b) $$\left(-\frac{2x^2}{3y}\right)^4 = \frac{(-2x^2)^4}{(3y)^4} = \frac{(-2)^4(x^2)^4}{3^4y^4} = \frac{16x^8}{81y^4}$$

- c) $$\left(\frac{x^3}{y^5}\right)^4 = \frac{(x^3)^4}{(y^5)^4} = \frac{x^{12}}{y^{20}}$$ **Ex. 59-66**

All summed up!

- Look to page 261 of your text for ALL these power rules to date summarized.

Rules for Nonnegative Integral Exponents

If a and b are nonzero real numbers, and m and n are nonnegative integers, then

1. $a^m a^n = a^{m+n}$ — Product rule for exponents
2. $\dfrac{a^m}{a^n} = a^{m-n}$ — Quotient rule for exponents ($m \geq n$)
3. $(a^m)^n = a^{mn}$ — Power of a power rule
4. $(ab)^n = a^n b^n$ — Power of a product rule
5. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$ — Power of a quotient rule

Section 4.1 With your OWN hand

- Definitions Q1-6
- Product rule Q7-18
- Zero exponents Q19-28
- Quotient Rule Q29-40
- Power Rule Q41-50
- Power of Product Q51-58
- Power of Quotient Rule Q59-66
- Simplify random stuff Q67-88
- Wordy Problems Q89-96

Remember… tan lines are ones with homework or group work problems in them

Section 4.2 Negative exponents (I've let you see them already- don't tell anyone)

- 1/x is the same as x^{-1}

- So Negative Integral Exponents are defined as

$$a^{-n} = \frac{1}{a^n}$$

What is one again?

- $a^{-n} * a^{n} = a^{-n+n} = a^{0} = 1$

Example 1 page 265

- a) $$2^{-5} = \frac{1}{2^5} = \frac{1}{32}$$
- b) $$(-2)^{-5} = \frac{1}{(-2)^5} = -\frac{1}{32}$$
- c) $-9^{-2} = -(9^{-2}) = -1/9^2 = -1/81$
- d) $$\frac{2^{-3}}{3^{-2}} = 2^{-3} \div 3^{-2} = \frac{1}{2^3} \div \frac{1}{3^2} = \frac{1}{8} \div \frac{1}{9} = \frac{1}{8} \cdot \frac{9}{1} = \frac{9}{8}$$

Ex. 7-16

Some pitfalls

- Watch the negative sign…
- $-5^{-2} = -(5^{-2})$ so the answer is –1/25

- Also if you see this… make it simpler

$$\frac{1}{3^{-2}} = 3^2$$

Helpful rules (pg 260) with negative powers…

- a must be non-zero

$$a^{-n} = \left(\frac{1}{a}\right)^n$$

$$a^{-1} = \frac{1}{a}$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

$$\frac{1}{a^{-n}} = a^n$$

Example 2 page 266

- a) $$\frac{2}{10^{-3}} = 2 \cdot 10^{3} = 2 \cdot 1000 = 2000$$
- b) $$\frac{2y^{-8}}{x^{-3}} = 2 \cdot y^{-8} \cdot \frac{1}{x^{-3}} = 2 \cdot \frac{1}{y^{8}} \cdot x^{3} = \frac{2x^{3}}{y^{8}}$$
- c) $$10^{-1} + 10^{-1} = \frac{1}{10} + \frac{1}{10} = \frac{2}{10} = \frac{1}{5}$$
- d) $$\left(\frac{3}{4}\right)^{-3} = \left(\frac{4}{3}\right)^{3} = \frac{64}{27}$$

Ex.17-26

Rules for Integral Exponents

- Just like positive exponents, if you multiply the numbers, add the exponents!

- $x^{-2} * x^{-3} = x^{-2+(-3)} = x^{-5}$

- Or with division… $\frac{y^3}{y^5} = y^3 y^{-5} = y^{-2}$

See Page 267 for MORE summary rules!

Rules for Integral Exponents

If a and b are nonzero real numbers, and m and n are integers, then

1. $a^{-n} = \frac{1}{a^n}$ — Definition of negative exponent
2. $a^{-1} = \frac{1}{a}, \frac{1}{a^{-n}} = a^n, a^{-n} = \left(\frac{1}{a}\right)^n, \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ — Negative exponent rules
3. $a^0 = 1$ — Definition of zero exponent
4. $a^m a^n = a^{m+n}$ — Product rule for exponents
5. $\frac{a^m}{a^n} = a^{m-n}$ — Quotient rule for exponents
6. $(a^m)^n = a^{mn}$ — Power of a power rule
7. $(ab)^n = a^n b^n$ — Power of a product rule
8. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ — Power of a quotient rule

Example 3 Working it out… page 268

- a) $b^{-3}b^{5} = b^{5-3} = b^{2}$
- b) $-3x^{-3} * 5x^{2} = -15x^{2-3} = -15x^{-1} = -15/x$
- c) $\dfrac{m^{-6}}{m^{-2}} = m^{-6-(-2)} = m^{-6+2} = m^{-4} = \dfrac{1}{m^{4}}$
- d) $\dfrac{4x^{-6}y^{5}}{-12x^{-6}y^{-3}} = \dfrac{x^{-6-(-6)}y^{5-(-3)}}{-3} = \dfrac{x^{0}y^{8}}{-3} = \dfrac{y^{8}}{3}$

Ex. 27-42

Example 4 page 268

- a) $(a^{-3})^2 = a^{-3*2} = a^{-6}$

- b) $(10x^{-3})^{-2} = 10^{-2}(x^{-3})^{-2} = 10^{-2}x^{6} = x^{6}/100$

- c)

$$\left(\frac{4x^{-5}}{y^{2}}\right)^{-2} = \frac{4^{-2}x^{10}}{y^{-4}} = \frac{x^{10}y^{4}}{4^{2}} = \frac{x^{10}y^{4}}{16}$$

Ex. 43-58

Scientific Notation… or “how I learned to love large numbers”

- We look at big numbers like 100000. and need to tell people later how many zeros we took out (who wants to write that many zeros anyway?).
- 1.00000 we ‘hopped’ the decimal place 5 hops to the left (positive) direction
- We write this as 1×10^5

Small numbers…

- 0.0000000003

- We need to hop the decimal place to the right (negative direction) and place it to the right of the first integer.
- 0x0000000003. We took 10 hops to the right
- We code it as 3×10^{-10}

nice examples

- $10(5.32) = 53.2$
- $10^2(5.32)=100(5.32)=532$
- $10^3(5.32)=1000(5.32)=5320$

and

- $10^{-1}(5.32)=.1(5.32)=.532$
- $10^{-2}(5.32)=.01(5.32)=.0532$
- $10^{-3}(5.32)=.001(5.32)=.00532$

Example 6 page 270

- Write in standard notation
- a) $7.02 \times 10^6 = 7020000 = 7,020,000$

- b) $8.13 \times 10^{-5} = .0000813$

Ex. 65-72

Example 7 page 271

- Write in Scientific Notation

a) 7346200 it's bigger than 10 so the exponent will be positive

7.3462×10^6

b) 0.0000348 it's less than 10 so the exponent will be negative

3.48×10^{-5}

c) 135×10^{-12} it should start with 1.35 so we need to go *positive* 2 places changing it to 1.35×10^{-10}

Ex. 73-80

Example 8 Computing with scientific notation (more exponents) page 272

- a) $(3x10^6)(2x10^8) = 3*2*10^6*10^8=6x10^{14}$

- b)

$$\frac{4x10^5}{8x10^{-2}} = \frac{4}{8}\cdot\frac{10^5}{10^{-2}} = \frac{1}{2}\cdot 10^{5-(-2)} = (0.5)(10^7) = 5(10^{-1})(10^7) = 5x10^6$$

Example 8c

- $(5x10^{-7})^3 = 5^3(10^{-7})^3 = 125\ (10^{-21}) = 1.25(10^2)(10^{-21}) =$

  1.25×10^{-19}

Ex. 81-92

Example 9 page 272
First Sci. Note. then math

- a) (3,000,000)(0.0002)

$=3x10^{6} \cdot 2x10^{-4} = 6x10^{2}$

b) $(20,000,000)^{3}(0.0000003)$

$=8x10^{21} \cdot 3x10^{-7}$

$= 24x10^{14}$ → need to move decimal 24. to 2.4 which is one to the right, or bigger!

$=24x10^{15}$ **Ex. 93-100**

More pen to paper Section 4.2

- Definitions Q1-6
- Get rid of negative exponents Q7-26
- Write numbers in standard notation Q27-58
- Present Value Formula Q59-64
- Scientific Notation Q65-80
- Computations with S.N. Q81-108
- Word problems Q109-116 (Learning Team)

Section 4.3

- Polynomials…You have already seen them in Chapter 1 and 2 so don’t panic!

Poly-want a cracker?

- What is a *term*?
- $4x^3$
- $-x^2y^3$
- 6ab
- -2
- $xyza^3$

A polynomial is a set of those terms

- A scrapbook of polynomials:
- x^2+5x+3
- $4x^3+10x^2+2x+100$
- $x+4$

Simplify – the standard way

- $4x^3+x+(-15x^2)+(-2)$
- We like to get rid of ( )'s
- We like them in order of decreasing exponent and alphabetized if possible
- The above becomes $4x^3-15x^2+x-2$
- $a^2b + b^2a + b^3a^2+ a^3b^2$ simplifies to…
- $a^3b^2 + b^3a^2 + a^2b + b^2a$

The degree of the polynomial

- We label the *terms* by the number in the exponent
- $4x^3 - 15x^2 + x - 2$
- 3rd order term (or degree term)
- 2nd order term (or degree term)
- 1st order term (or degree term)
- 0th order term (or degree term)
  this is also called a constant
  [Try this on a calculator… if the power of the x with the constant is zero, then it is x^0

You are constantly…

- [Try this on a calculator… if the power of the x with the constant is zero, then it is x^0
- What is 6^0? Or 1^0? Or even 1000^0?
- So what is $10 * x^0$?

What's in a coefficient?

- $4x^3-15x^2+x-2$
- The third order term's coefficient is 4
- The second order term's coefficient is –15
- The first order term's coefficient is 1
- The constant is –2 (the coefficient to the zeroth order term)

Example 1 Got Coefficient? pg277

- What are the coefficients of x^3 and x^2 in each:
- a) x^3+5x^2-6 which is $\_x^3+5x^2-6$
- 1 and 5
- b) $4x^6- x^3 + x$ is the same as $4x^6- x^3 + 0x^2 + x$
- So it's –1 and 0 **Ex. 7-12**

Again… how we *like* to order them

- We don't like $-4x^2+1+5x+x^3$
- We do like x^3-4x^2+5x+1
- The coefficient of the highest order term (x^3) is called the leading coefficient
  - The leading coefficient is 1 in this case
- The order of the polynomial is the exponent (or power) of the HIGHEST term

Special Definitions

- A *monomial* has only one term
- x, x^2, 3
- A binomial has two terms
- $x+5$, or x^3+4
- A trinomial has three terms (see a trend?)
- $10x^4+6x+100$

Example 2 pg 277

- Identify each as a polynomial as either a monomial, binomial or trinomial and state it's degree…
- a) $5x^2-7x^3+2$ → is a trinomial of 3rd order (degree)
- b) $x^{43}-x^2$ → is a binomial of 43rd order (degree)
- c) $5x=5x^1$ → is a polynomial of degree 1 (order)
- d) –12 → is a monomial of degree 0 (order)
- **Ex. 13-24**

Example 3 plug in the number pg 278

- The value of a polynomial…
- a) Find the value of $-3x^4-x^3+20x+3$ when $x=1$ →
  $-3(1)^4-(1)^3+20(1)+3 =$
  $-3-1+20+3 = 19$
- b) The same equation when $x=-2$ →

$-3(-2)^4-(-2)^3+20(-2)+3 = -48+8-40+3 = -77$

Ex. 25-32

Example 4 another look pg279

a) and if $P(x) = -3x^4 - x^3 + 20x + 3$ and you're told to find P(1)

It's the same as 3a)

$-3(1)^4 - (1)^3 + 20(1) + 3 = -3 - 1 + 20 + 3 = 19$

b) If $D(a) = a^3 - 5$ find D(0), D(1), D(2)

$0^3 - 5 = -5$, $1^3 - 5 = 1 - 5 = -4$, $2^3 - 5 = 8 - 5 = 3$

Ex. 33-38

Adding Polynomials

- To add two polynomials, add the like terms

- (You have done all this already as well!)

Example 5 pg 279

- a) $(x^2-6x+5)+(-3x^2+5x-9)$
- Group the terms together
- $x^2-3x^2-6x+5x+5-9$
- Then add the LIKE terms
- $-2x^2-x-4$

Example 5b

- Or you can add them vertically (like we did for the addition method of finding solutions)
- $(-5a^3+3a-7)+(4a^2-3a+7)$

$$
\begin{array}{rrrr}
-5a^3+ & & 3a & -7 \\
 & 4a^2 & -3a & +7 \\
\hline
-5a^3+ & 4a^2 & +0 & +0
\end{array}
$$

Ex. 39-52

Subtraction of Polynomials

- subtract (stuff b) from (stuff a)
- This is the same as saying (stuff a) – (stuff b)
- You have to multiply everything in stuff b by –1
- like -1*($4a^2$-3a+7) which is $-4a^2+3a-7$

Example 6a pg 280

- Perform the following:
- a) $(x^2-5x-3) – (4x^2+8x-9)$
- So this is $x^2-5x-3-4x^2-8x+9$
- $x^2 -4x^2 -5x -8x -3+9$
- $-3x^2 -13x +6$

Example 6b

- Or do it vertically…

$4y^3 \quad -3y \quad +2$

$- \quad (5y^2 \; - 7y \quad -6)$

- Becomes

$4y^3 \quad -3y \quad +2$

$+ \quad -5y^2 \; +7y \quad +6$

6b the end

$$
\begin{array}{rrrr}
4y^3 & & -3y & +2 \\
+ & -5y^2 & +7y & +6 \\
\hline
4y^3 & -5y^2 & +4y & +8
\end{array}
$$

Just make sure you line up like terms (terms are defined by the power of the exponents)

- **Ex. 53-66**

Example 7

Handling the mess…

$(2x^2-3x)+(x^3+6)-(x^4-6x^2-9)$ →

$2x^2-3x+x^3+6-x^4+6x^2+9$ →

$-x^4 +x^3 +6x^2 +2x^2-3x+6+9$ →

$-x^4 +x^3 +8x^2-3x+15$

Ex. 83-90

Trying your hand at it… Sect. 4.3

- Definitions Q 1- Q6
- Naming the coefficients of x^3 and x^2 terms Q 7-12
- Monomial? Binomial? Trinomial? Degree (order)? Q13-24
- Evaluate at the given value Q25-38
- Perform the indicated operation + Q39-Q52
- Perform the indicated operation – Q53-66
- Add vertically Q67-74
- Some standard problems Q75-90
- Word problems Q91-104   Learning Team

Section 4.4 Multiplying Polynomials

- If you multiply two monomials, you are doing stuff you already know!
- $x^3 = x*x*x$ and $x^5 = x*x*x*x*x$
- How many x's? … 8.
- So it's x^8
- But you could have already said:

  $x^3 * x^5 = x^{3+5} = x^8$

The Product Rule

- Another one for the index card…
- If a is any real number and m and n are any positive integers, then

$$a^m * a^n = a^{m+n}$$

Example 1 – find the products pg285

- a) $2x^3 * 3x^4 = 6x^{3+4} = 6x^7$
- b) $(-2ab)(-3ab) = 6a^{1+1}\, b^{1+1} = 6a^2b^2$
- d) $(3a^2)^3 = 3^3(a^2)^3 = 27a^6$
- **Ex. 7-22**

Example 2 page 285

- a) $3x^2(x^3-4x) = 3x^2 \cdot x^3 - 3x^2 \cdot 4x = 3x^5-12x^3$

- b) $(y^2-3y+4)(-2y) = y^2(-2y)-3y(-2y)+4(-2y)$

$= -2y^3+6y^2-8y$

- c) $-a(b-c)= (-a)b-(-a)(c) = -ab+ac=ac-ab$

Ex. 23-36

Example 3 Remember the Alamo? Remember the distributive property? page 286

- a) $(x+2)(x+5)$
- First times first
- plus first times second
- plus second times first
- plus second times second
- $x*x + x*5 + 2*x + 2*5$
- $x^2 + 5x+2x+10 = x^2 + 7x +10$

One more step for mankind…
Example 3b

- $(x+3)\ (x^2+2x-7)$
- $(x+3)x^2 + (x+3)2x + (x+3)(-7)$ see it?
- $x^3+3x^2 +2x^2+6x-7x-21$ add the like terms
- $x^3+5x^2 -x-21$

- **Ex. 37-48**

Bonus Example - Goin' Vertical

- You can also multiply things vertically
- (x-2) (3x+7)

$$
\begin{array}{r}
3x+7 \\
*\quad x-2 \\
\hline
-6x-14 \\
+3x^2+7x \\
\hline
3x^2+x-14
\end{array}
$$

another one

- $(x+2)(x^2-x+1)$

x^2-x+1

$x+2$

$2x^2-2x+2$

$x^3- x^2 +x$

----

x^3+x^2-x+2

The multiplication of polynomials

- The rule:

- To multiply polynomials, multiply each term of one polynomial by every term of the other polynomial, then combine like terms.

The opposite of a polynomial

- The opposite of y is –y
  - because y-y =0
- The opposite of x^2-3x+1 is $-x^2+3x-1$ because if you add these, they kill each other term by term and = 0
- -(a-b) is -a+b
- -(a+b) is -a-b

Example 5 Opposites are negative page 287

- Find the opposite of each polynomial…
- a) opposite of $x-2$ → $-(x-2) = -x+2$
- b) opposite of $9-y^2$ → $-(9-y^2) = -9 + y^2$
- c) opposite of $a+4$ → $-(a+4) = -a-4$
- d) opposite of
  $–x^2+6x-3$ → $-(–x^2+6x-3)$ → x^2-6x+3

Ex. 49-56

Section 4.4 Trying it on for size

- Definitions Q1-Q6
- Find each product Q7-Q48
- Find the opposite Q49-56
- Perform the operation indicated Q57-76
- Word Problem Q77-88

Section 4.5 Multiplication of Bionomials

- Curses! FOILed again.
- With binomials you've played with (tonight) first to first, first to second, second to first, and second to second.
- But FOIL helps you remember this method
- [These look like $(x+4)(x^2+x)$]

FOIL is the same thing with a flashy name

- F First terms $(x+4)(x^2+x)$
- O then the Outer terms $(x+4)(x^2+x)$
- I then the inner terms $(x+4)(x^2+x)$
- L then the Last $(x+4)(x^2+x)$

Example 1 Aluminum FOIL pg291

- a) $(x+2)(x-4) = x^2-4x+2x-8 = x^2 - 2x - 8$

F O I L

b) $(2x+5)(3x-4) = 6x^2-8x+15x-20 = 6x^2+7x-20$

F O I L

c) $(a-b)(2a-b) = 2a^2-ab-2ab+b^2 = 2a^2-3ab+b^2$

F O I L

d) $(x+3)(y+5) = xy + 5x + 3y + 15$ done!

F O I L

Ex. 5-28

Example 2 Tin Foil pg 291

- FOIL works for any two binomials…
- a) $(x^3-3)(x^3+6) = x^6 + 6x^3 - 3x^3 - 18 = x^6 + 3x^3 - 18$

F O I L

b) $(2a^2+1)(a^2+5) = 2a^4 + 10a^2 + a^2 + 5 = 2a^4 + 11a^2 + 5$

F O I L

Ex. 29-40

Example 3 Quick FOIL pg 292

- You can usually add the two middle terms in your HEAD and just write down the answer!
- a) $(x+3)(x+4) = x^2+7x\ \ +12$

  F O+I L
- b) $(2x-1)(x+5) = 2x^2+4x\ \ -5$

  F O+I L
- c) $(a-6)(a+6) = a^2-36$ there is a 0a in there!

  F L O+I

Ex. 41-66

Example 4 Fractional FOIL pg.292

(a) $(x-1)(x+3)(x-4) = (x^2+2x-3)(x-4) =$

$x(x^2+2x-3) - 4(x^2+2x-3) =$

$x^3+2x^2-3x-4x^2-8x+12 =$

$x^3-2x^2-11x+12$

(b) $(\frac{1}{2}X - 2)(\frac{1}{3}x + 1) = \frac{1}{6}x^2 - \frac{2}{3}x + \frac{1}{2}x - 2 = \frac{1}{6}x^2 - \frac{1}{6}x - 2$

Ex. 67-74

Section 4.5 Just do it!

- Definitions Q1-Q4
- Using FOIL Q5-40
- Using quick FOIL Q41-66
- Messier FOILs   Q67-96
- Word problems Q97-102

Section 4.6 Special Products (short cuts you can take!)

- Some problems are not problems at all… they are gifts.

- Sorry about that…

Special Products

- Every time you see the same type of problem, you can use the same tricks!

The Square of a Binomial

- It looks like $(a+b)^2$
- working it out…
- $(a+b)(a+b) = a^2+ab+ab+b^2 = a^2+2ab+b^2$

- It ALWAYS turns out this way!

For your index card…

- $(a+b)(a+b) = = a^2+2ab+b^2$

Example 1 pg 296

- a) $(x+3)^2 = x^2+2(x)(3) + 3^2 = x^2+6x+ 9$

- b) $(2a+5)^2 = (2a)^2 +2(2a)(5) + 5^2 =$ $4a^2+20a+25$

- When doing this… students often forget the 2ab “middle” term. This is the pitfall you need to watch out for!
- **Ex. 7-22**

What’s the Difference?

- What about the special problem $(a-b)^2$?
- It’s the same but for ONE minus sign
- $a^2 - 2ab + b^2$

Example 2 Don't get negative pg 297

- $(x-4)^2 = x^2 - 2(x)(4) + 4^2 = x^2 - 8x + 16$

- $(4b-5y)^2 = (4b)^2 - 2(4b)(5y) + (-5y)^2 =$
- $16b^2 - 40by + 25y^2$

Ex. 23-36

Mix them in a blender…

- What about $(a+b)(a-b)$?
- The CENTER term $-ab$ $+ab$ kill each other!
- It always becomes a^2-b^2

Example 3 Kill the middle! pg 297

- $(x+2)(x-2) = x^2-4$
- $(b+7)(b-7) = x^2-49$
- $(3x-5)(3x+5) = 9x^2-25$
- **Ex. 37-48**

What if the binomial is taken to HIGHER powers? Break it down.

- Example 4 page 298
- $(x+4)^3=$
- $(x+4)^2(x+4) =$
- $(x^2+8x+16)(x+4)=$ then expand on x and 4
- $(x^2+8x+16)x + (x^2+8x+16)4$ and multiply it out and gather the like terms…
- $x^3+12x^2+48x+64$

Example 4b

- $(y-2)^4 = (y-2)^2(y-2)^2 =$
- $(y^2-4y+4)\ (y^2-4y+4) =$
- $(y^2-4y+4)\ y^2 + (y^2-4y+4)\ (-4y) + (y^2-4y+4)\ 4 =$
- $y^4-4y^3+4y^2-4y^3+16y^2 - 16y+4y^2 - 16y+16$
- $y^4-8y^3+24y^2-32y+16$

Ex. 49-56

Section 4.6 Practices!

- Definitions again! Q1-6
- Square positive binomials Q7-22
- Square negative binomials Q23-36
- Find products (pos and negs) Q37-48
- Expanding binomials to higher powers Q49-56
- Mixed bag o problems Q51-80
- Word probs… Q81-92   Learning Team

Section 4.7 Division
“One binomial, under God, indivisible, …”

- Sure we can divide binomials!
  They are just numbers in disguise!
- But do we want to?
- Sure!

- (I shouldn’t have asked that question)

Remember division can be hazardous!

$a \div b = c$ where $b \neq 0$

as long as $c \bullet b = a$

Definitions

- In $a \div b = c$ a is called the dividend and b is the divisor

$$x^5 / x^2 = \frac{x\,x\,x\,x\,x}{x\,x} = \frac{x\,x\,x}{1} = x^3$$

And upside down?

$$x^2 / x^5 = \frac{x\,x}{x\,x\,x\,x\,x} = \frac{1}{x\,x\,x} = \frac{1}{x^3}$$

which can also be written as x^{-3}

Quotient Rule

- Suppose a is not ZERO.

If $m \geq n$ then $$\frac{a^m}{a^n} = a^{m-n}$$

If n>m, then $$\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$$

What is the power of one? Or zero really?

- x^4 divided by x^4
- $x^{4-4} = x^0 = 1$!
- So $a^0 = 1$

Extra example: some zero powers

- a) $5^0 = 1$
- b) $(3xy)^0 = 1$
- c) $a^0+b^0 = 1+1=2$

Dividing using the quotient rule Example 1 page 302

- a) $(12x^5)/(3x^2) = \dfrac{-12x^5}{2x^3} = 4x^{5-2} = 4x^3$
- b) $\dfrac{-4x^3}{2x^3} = -2x^{2-3} = 2x^0 = -2 \bullet 1 = -2$
- c) $\dfrac{-10a^3b^4}{-2a^2b^2} = 5a^{3-2}b^{4-2} = 5ab^2$

Dividing a Polynomial by a Monomial

- Break it up, then handle the parts!

Example 2a pg 303

$$\frac{5x-10}{5}=\frac{5x}{5}-\frac{10}{5}=x-2$$

Example 2b

- Find the quotient for $(-8x^6+12x^4-4x^2)$ divided by $4x^2$

$$\frac{-8x^2+12x^4-4x^2}{4x^2}=\frac{-8x^6}{4x^2}+\frac{12x^4}{4x^2}-\frac{4x^2}{4x^2}=-2x^4+3x^2-1$$

- **Ex. 25-32**

Dividing a Polynomial by a Binomial – Looong division

- Rememer?

```
      36   .
7 |  253
     21
       43
       42
        1
```

Amazingly, we can do this with polynomials as well…

- Divide x^2-3x-10 by x+2

$$
\begin{array}{r l}
 & \underline{x \quad .} \\
x+2 \,| & x^2 - 3x - 10 \\
 & \underline{x^2 + 2x} \\
 & \quad -5x
\end{array}
$$

Next step

$$
\begin{array}{r l}
 & \underline{x - 5 \quad .} \\
x+2 \,| & x^2 - 3x - 10 \\
 & \underline{x^2 + 2x} \\
 & \quad -5x - 10 \\
 & \quad \underline{-5x - 10} \\
 & \qquad\quad 0
\end{array}
$$

Example 3 page 304

Divide x^3-5x-1 by $x-4$ and state the remainder

$$
\begin{array}{l}
\qquad\ \ \underline{x^2\ +4x+11\ \ .} \\
x-4\ |\ x^3+0x^2-5x-1 \\
\qquad\ \underline{x^3-4x^2} \\
\qquad\qquad 4x^2\ -5x \\
\qquad\qquad \underline{4x^2\ -16x} \\
\qquad\qquad\qquad 11x-1 \\
\qquad\qquad\qquad \underline{11x\ -44}
\end{array}
$$

43 which is the remainder

Ex. 33-36

Example 4 pg 305

Divide $2x^3-7x^2+0x-4$ by $2x-3$ and state the remainder

$$
\begin{array}{r|l}
 & \underline{x^2-2x-3\quad .} \\
2x-3 & 2x^3-7x^2+0x-4 \\
 & \underline{2x^3-3x^2} \\
 & \quad -4x^2+0x \\
 & \quad \underline{-4x^2+6x} \\
 & \qquad\quad -6x-4 \\
 & \qquad\quad \underline{-6x+9} \\
 & \qquad\qquad\quad -13
\end{array}
$$

-13 which is the remainder

Ex. 37-50

What to do with what remains?

- Start with the $\frac{\text{dividend}}{\text{divisor}} = \text{quotient} + \frac{\text{remainder}}{\text{divisor}}$

For example (in *english)*

$$\frac{19}{5} = 3 + \frac{4}{5}$$

So the example above becomes

- $x^2-2x-3 -13/(2x-3)$ Yuck!

Example 5 is the same thing but simpler… page 306

- Express $\dfrac{-3x}{x-2}$ as quotient and remainder

```
          -3    .
x-2 |  -3x+0
        -3x+6
           -6
```

So the answer is $-3 + \dfrac{-6}{x-2}$ **Ex. 51-66**

Section 4.7 Divide them!

- Definitions Q1-6
- Find the quotient Q7-32
- Use long division Q33-50
- Using the remainder Q51-66
- Random quotients Q67-86
- 'Real World' problems Q87-91 LT's!