MTH 209
The University of Phoenix

Chapter 6

Operations with Rational Expressions

And now for something…

- Easier!
- This is like MTH208 material, but with variables added…

Section 6.1 Reducing Rational Expressions a review of factoring and reducing

- In number world, it looks like this:

$$\frac{3}{4}, \frac{-9}{-6}, 7, 0, 4, \frac{1}{2}$$

- In polynomial word, it looks like this:

$$\frac{x^2-1}{x+8}, \frac{3a^2+5a-3}{a-9}, \frac{3}{7}, w$$

Example 1 page 378
Evaluating a rational expression

- Given x=-3 what is $\frac{4x-1}{x+2} = \frac{4(-3)-1}{-3+2} = \frac{-13}{-1} = 13$

- Find R(4) if $R(x) = \frac{3x+2}{2x-1} = \frac{3(4)+2}{2(4)-1} = \frac{14}{7} = 2$

** Ex. 7-12**

Example 2 pg 379
What CAN'T x be?

Remember, 0 on the bottom = explosion.

- a) $\frac{x^2 - 1}{x + 8}$ x=-8 = death!
- b) $\frac{x + 2}{2x + 1}$ x= - ½ = death!
- c) $\frac{x + 5}{x^2 - 4}$ x= 2 OR –2 = death!

** Ex. 13-20**

Example 3 pg 379
What CAN x be?

Remember, 0 on the bottom = explosion.

- a) $\dfrac{x^2-9}{x+3}$ if x=-3 death – so everything else
- b) $\dfrac{x}{x^2-x-6}$ Solve quadratic on bottom
  (x-3)(x+2)=0 so x=3 or x=-2 = death
  So everything else.
- c) $\dfrac{x-5}{4}$ No death possible – all numbers work

** Ex. 21-28**

The domain of answers

- The answers that WILL work in the above equations include ALL rational numbers EXCEPT those we found make it blow up (zero in the denominator).

The number example of Reducing things to their Lowest Terms

- We can take ANY of the fractions and reduce them to the first one:

$$\frac{3}{5} = \frac{6}{10} = \frac{9}{15} = \frac{12}{20} = \frac{15}{25}$$

$$\frac{3}{5}\frac{2}{2} = \frac{6}{10}, \frac{3}{5}\frac{3}{3} = \frac{9}{15}, etc.$$

So going backwards…

- We separate (factor) out like terms top and bottom, then cancel them.

$$\frac{6}{10} = \frac{2\cdot 3}{2\cdot 5} = \frac{3}{5}$$

Isn't this a nice step backwards… catch a breather!

- Warning, of course this does NOT work with addition or subtraction!

$$\frac{6}{10} = \frac{2+4}{2+8}$$

- You can't touch the 2's here!

So reducing fractions looks like:

- If a $\neq 0$ and c $\neq$ 0 then:

$$\frac{ab}{ac} = \frac{b}{c}$$

The Reducing Diet

- 1) Factor the numerator and denominator completely.
- 2) Divide the numerator and denominator by the greatest common factor (kill the like numbers top and bottom).

Example 3 page 381

- Reduce to lowest terms:

- a) $$\frac{30}{42} = \frac{2 \cdot 3 \cdot 5}{2 \cdot 3 \cdot 7} = \frac{5}{7}$$

- b) $$\frac{x^2 - 9}{6x + 18} = \frac{(x-3)(x+3)}{6(x+3)} = \frac{x-3}{6}$$

Example 3c

c)

$$\frac{3x^2+9x+6}{2x^2-8}=\frac{3(x+2)(x+1)}{2(x+2)(x-2)}=\frac{3x+3}{2(x-2)}$$

** Ex. 29-52**

Reducing by the Quotient Rule

- Suppose a is not ZERO.

If $m \geq n$ then $$\frac{a^m}{a^n} = a^{m-n}$$

If n>m, then $$\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$$

Example 4 page 382
Using the Quotient Rule

- a) $$\frac{3a^{15}}{6a^{7}} = \frac{3a^{15}}{3 \cdot 2a^{7}} = \frac{a^{15-7}}{2} = \frac{a^{8}}{2}$$

- b) $$\frac{6x^{4}y^{2}}{4xy^{5}} = \frac{2 \cdot 3x^{4}y^{2}}{2 \cdot 2xy^{5}} = \frac{3x^{4-1}}{2y^{5-2}} = \frac{3x^{3}}{2y^{3}}$$

** Ex. 53-64**

Example 5 page 382

- Reduce 420/616 to it's lowest terms…

$$2\overline{)420}\quad 2\overline{)210}\quad 3\overline{)105}\quad 5\overline{)35}\quad 7$$

$$2\overline{)616}\quad 2\overline{)308}\quad 2\overline{)154}\quad 7\overline{)77}\quad 11$$

$$\frac{420}{616}=\frac{2^2\cdot 3\cdot 5\cdot 7}{2^3\cdot 7\cdot 11}=\frac{3\cdot 5}{2\cdot 11}=\frac{15}{22}$$

** Ex. 65-72**

Another neat shortcut… What equals –1?

- If you divide

$$\frac{a-b}{b-a}=\frac{-(b-a)}{(b-a)}=\frac{-1(b-a)}{(b-a)}=-1$$

Let's use THAT trick in Example 6 page 383

- a) $$\frac{5x-5y}{4y-4x}=\frac{5(x-y)}{4(y-x)}=\frac{5(-1)}{4}=-\frac{5}{4}$$

- b)

$$\frac{m^2-n^2}{n-m}=\frac{(m-n)(m+n)}{n-m}=-1(m+n)=-m-n$$

** Ex. 73-80**

Another quick caution

- We now know: $$\frac{a-b}{b-a}=-1$$
- But we can't work with: $$\frac{a-b}{a+b}=itself$$
- It has no common factors. It just IS.

Factoring out the opposite of the Common Factor

- Translation: Take out a negative sign from everything.
- $-3x-6y$ we can take out 3 → $3(-x-y)$
- or we can take out –3 → $-3(x+y)$
- Easy?

Example 7 page 384, taking out the negative (attitude)

- Factor to lowest terms:

$$\frac{-3w-3w^2}{w^2-1}=\frac{-3w(1+w)}{(w-1)(w+1)}=\frac{-3w}{w-1}=\frac{3w}{1-w}$$

- You don't always have to do the last step, but it makes it look nicer.

** Ex. 81-90**

Putting the steps all together

- 1) Reducing is done by dividing out all common factors
- 2) Factor the numerator and denominator completely to see the common factors
- 3) Use the quotient rule to reduce a ratio of two monomials
- 4) You may have to factor out common factor with a negative sign to get identical factors in the numerator and denominator.
- 5) The quotient of a-b and b-a is –1 (a helpful trick)

The review section on factoring and reducing Section 6.1

We'll only pause on these problems if you feel we need to … class poll.

- Definitions Q1-6
- Evaluating each rational expression Q7-28
- Reducing to lowest terms Q29-52
- Reducing with quotient rule for exponents Q53-72
- Dividing by a-b and b-a Q73-80
- Factoring out the opposite of a common factor Q81-112
- Word problems Q113-120

6.2 Multiplication and Division

- If b and d are not = zero… then

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

Example 1 page 389

- Find the product

$$\frac{6}{7}\cdot\frac{14}{15}=\frac{84}{105}=\frac{21\cdot 4}{21\cdot 5}=\frac{4}{5}$$

** Ex. 5-12**

Multiplying Rational Expressions Example 2a and b page 389

- Find the products

$$\frac{9x}{5y}\cdot\frac{10y}{3xy}=\frac{3\cdot 3\cdot x\cdot 2\cdot 5\cdot y}{5\cdot y\cdot 3\cdot x\cdot y}=\frac{6}{y}$$

$$\frac{-8xy^4}{3z^3}\cdot\frac{-15z}{2x^5y^3}=\frac{-2\cdot 2\cdot 2xy^4\cdot 3\cdot 5\cdot z}{3z^3\cdot 2\cdot x^5\cdot y^3}=\frac{-20xy^4z}{z^3x^5y^3}=\frac{-20y}{z^2x^4}$$

** Ex. 13-22**

Going beyond monomials… multiplying rational expressions Ex 3 page 390 ** Ex. 23-30**

- a) $\dfrac{2x-2y}{4}\cdot\dfrac{2x}{x^2-y^2}=\dfrac{2(x-y)\cdot 2\cdot x}{2\cdot 2\cdot (x-y)(x+y)}=\dfrac{x}{x+y}$

- b) $\dfrac{x^2+7x+12}{2x+6}\cdot\dfrac{x}{x^2-16}=\dfrac{(x+3)(x+4)\cdot x}{2(x+3)(x-4)(x+4)}=\dfrac{x}{2(x-4)}=\dfrac{x}{2x-8}$

- c) $\dfrac{a+b}{6a}\cdot\dfrac{8a^2}{a^2+2ab+b^2}=\dfrac{(a+b)\cdot 2\cdot 4a^2}{2\cdot 3a\cdot (a+b)^2}=\dfrac{4a}{3(a+b)}=\dfrac{4a}{3a+3b}$

Divide? No flip and multiply!

- Remember this blast from the past?

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

Example 4 page 390

- a) $$5 \div \frac{1}{2} = 5 \cdot 2 = 10$$
- b) $$\frac{6}{7} \div \frac{3}{14} = \frac{6}{7} \cdot \frac{14}{3} = \frac{2 \cdot 3 \cdot 2 \cdot 7}{7 \cdot 3} = 4$$

** Ex. 31-38**

Of course, you can do the same with expressions! Ex 5 page 391

Find each quotient!

- a) $$\frac{5}{3x} \div \frac{5}{6x} = \frac{5}{3x} \cdot \frac{6x}{5} = \frac{5 \cdot 2 \cdot 3x}{3x \cdot 5} = 2$$
- b) $$\frac{x^7}{2} \div \frac{2x^2}{1} = \frac{x^7}{2} \cdot \frac{1}{2x^2} = \frac{x^5}{4}$$
- c) $$\frac{4-x^2}{x^2+x} \div \frac{x-2}{x^2-1} = \frac{4-x^2}{x^2+x} \cdot \frac{x^2-1}{x-2} = \frac{(2-x)(2+x)(x+1)(x-1)}{x(x+1)(x-2)} =$$

$$-1\frac{(2+x)(x-1)}{x} = \frac{-1(x^2+x-2)}{x} = \frac{-x^2-x+2}{x}$$

** Ex. 39-52**

Example 6 pg 391 Now with clunky fraction/division bar ** Ex. 53-60**

- a) $$\frac{\frac{a+b}{3}}{\frac{1}{6}} = \frac{a+b}{3} \div \frac{1}{6} = \frac{a+b}{3} \cdot \frac{6}{1} = \frac{a+b}{3} \cdot \frac{2 \cdot 3}{1} = 2(a+b) = 2a+2b$$

- b) $$\frac{\frac{x^2-1}{2}}{\frac{x-1}{3}} = \frac{x^2-1}{2} \div \frac{x-1}{3} = \frac{x^2-1}{2} \cdot \frac{3}{x-1} = \frac{(x-1)(x+1) \cdot 3}{2(x-1)} = \frac{3x+3}{2}$$

- c) $$\frac{\frac{a^2+5}{3}}{2} = \frac{a^2+5}{3} \div \frac{2}{1} = \frac{a^2+5}{3} \cdot \frac{1}{2} = \frac{a^2+5}{6}$$

Section 6.2 Doings

- Definitions Q1-4
- Perform the operations with number fractions Q5-12
- Do it with variables Q13-30
- Just with numbers Q31-38
- Do it with polynomials Q39-60
- A mixed bag of divisions Q61-80
- Word problems Q81-88

Section 6.3 Finding the least common denominator

- AGAIN. You have done all of Ch 7 before… this should be be a good review still!

We’re going to Build You UP!

- Building up denominators…
- Covert the denominator to 21

$$\frac{2}{3} = \frac{2}{3} \cdot \frac{7}{7} = \frac{14}{21}$$

It's the same for a polynomial fraction

- Start with a fraction of $\dfrac{5}{x+3}$
- We want the denominator to be x^2-x-12
- First, factor the desired denominator

$x^2-x-12=(x+3)(x-4)$ so we need (x-4) on top and bottom

$$\frac{5}{x+3}\cdot\frac{x-4}{x-4}=\frac{5x-20}{x^2-x-12}$$

Example 1 pg 397– Building up denominators ** Ex. 5-24**

- a) $$3 = \frac{?}{12} = \frac{3}{1} \cdot \frac{12}{12} = \frac{36}{12}$$
- b) $$\frac{3}{w} = \frac{?}{wx} = \frac{3}{w} \cdot \frac{x}{x} = \frac{3x}{wx}$$
- c) $$\frac{2}{3y^3} = \frac{?}{12y^8} = \frac{2}{3y^3} \cdot \frac{4y^5}{4y^5} = \frac{8y^5}{12y^8}$$

Example 2 page 397
Or you might have to factor first, THEN build up the fraction

- a)

$$\frac{7}{3x-3y}=\frac{?}{6y-6x}=\frac{7}{3(x-y)}\cdot\frac{-2}{-2}=\frac{-14}{-6(x-y)}=\frac{-14}{-6x+6y}=\frac{-14}{6y-6x}$$

- b)

$$\frac{x-2}{x+2}=\frac{?}{x^2+8x+12}=\frac{?}{(x+2)(x+6)}=\frac{x-2}{x+2}\cdot\frac{(x+6)}{(x+6)}=\frac{(x-2)(x+6)}{x^2+8x+12}$$

** Ex. 25-36**

Back again to the LCD (not the LSD)

- We want to use the maximum number of factors that show up in either factored number.

- 24= 2*2*2*3= 2^3*3
- 30 = 2*3*5
- Multiply those together 2*2*2*3*5=120
- We have our LCD

Cooking with LCD

1. Factor the denominator completely.

(For clarity) Use exponent notation for repeated factors.

2. Write the product of all the different factors that appear together in the denominators.

3. Use the highest power you see in either list and multiply them all together.

Example 3 page 399

- Finding the LCD
- a) 20,50

$20=2^2*5$

$50=2*5^2$ → $2*2*5*5 = 100$

Ex 3b

x^3yz^2, x^5y^2z, xyz^5

x^3yz^2

x^5y^2z

xyz^5

$= x^5y^2z^5$

Ex 3c

- a^2+5a+6, a^2+4a+4

$a^2+5a+6= (a+2)(a+3)$

$a^2+4a+4= (a+2)^2$

$(a+2)^2(a+3)=$ and you could multiply it out if you needed to or call it quits here.

** Ex. 37-50**

Ex 4 page 399

Now doing what we've been doing in a real denominator.

- a) $\frac{4}{9xy}, \frac{2}{15xz}$

$9xy = 3^2xy$

$15xz = 3 * 5xz$ So the LCD = $3^2 5xyz$

So we get the first term needs a 5z/5z stuck to it, the second term needs a 3y/3y added to it.

$$\frac{4}{9xy} \cdot \frac{5z}{5z} = \frac{20z}{45xyz}$$

$$\frac{2}{15xz} \cdot \frac{3y}{3y} = \frac{6y}{45xyz}$$

DONE!

Ex 4b

- b) $\frac{5}{6x^2}, \frac{1}{8x^3y}, \frac{3}{4y^2}$
- $6x^2=2*3x^2$ this term needs $4xy^2$/ $4xy^2$
- $8x^3y=2^3x^3y$ this term needs 3y/ 3y
- $4y^2=2^2y^2$ this term needs $6x^3$/ $6x^3$
- So we want $2^3*3*x^3*y^2$
- next page…

Ex 4b continued ** Ex. 51-62**

$$\frac{5}{6x^2} = \frac{5 \cdot 4xy^2}{6x^2 \cdot 4xy^2} = \frac{20xy^2}{24x^3y^2}$$

$$\frac{1}{8x^3} = \frac{1 \cdot 3y}{8x^3 \cdot 3y} = \frac{3y}{24x^3y^2}$$

$$\frac{3}{4y^2} = \frac{3 \cdot 6x^3}{4y^2 \cdot 6x^3} = \frac{18x^3}{24x^3y^2}$$

And finally the LCD with polynomials (factor first!) pg 400

- Ex 5a) $\dfrac{5x}{x^2-4}, \dfrac{3}{x^2+x-6}$

x^2-4 = (x-2) (x+2)   so this needs (x+3)

x^2+x-6= (x-2) (x+3)   and this needs (x+2)

So our LCD is (x-2)(x+2)(x+3)

Ex 5 continued

$$\frac{5x}{x^2-4}=\frac{5x(x+3)}{(x-2)(x+2)(x+3)}=\frac{5x^2+15x}{(x-2)(x+2)(x+3)}$$

$$\frac{3}{x^2x-6}=\frac{3(x+2)}{(x-2)(x+3)(x+2)}=\frac{3x+6}{(x-2)(x+3)(x+2)}$$

** Ex. 63-74**

Denominator Exercising Section 6.3

- Definitions Q1-4
- Building up rational expressions Q5-Q24
- The same but with polynomials Q25-36
- Two numbers, what is the LCD Q37-50
- Find the LCD with fractions Q51-62
- Find LCD with expressions Q63-74
- Two more problems Q75-76

Section 6.4 Addition and Subtraction

- Now we add the *one more complication* of adding/subtracting and having to make the denominators match, but with now with more nutritious polynomials.

Addition and Subtraction of Rational Numbers

- If b is not zero…

$$\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}\ldots and \ldots \frac{a}{b}-\frac{c}{b}=\frac{a-c}{b}$$

Example 1 page 404

- Just by the numbers…
- a)

$$\frac{1}{12}+\frac{7}{12}=\frac{8}{12}=\frac{4}{4}\cdot\frac{2}{3}=\frac{2}{3}$$

- b)

$$\frac{1}{4}-\frac{3}{4}=\frac{-2}{4}=-\frac{1}{2}$$

** Ex. 5-12**

Example 2 page 404

- Do the sum or difference… $\frac{3}{20}+\frac{7}{12}$
- $20=2^2*5$
- $12=2^2*3$ So the LCD is 2^2*3*5 or 60

$$\frac{3}{20}+\frac{7}{12}=\frac{3\cdot 3}{20\cdot 3}+\frac{7\cdot 5}{12\cdot 5}=\frac{9}{60}+\frac{35}{60}=\frac{44}{60}=\frac{4\cdot 11}{4\cdot 15}=\frac{11}{15}$$

example 2b

- b) $\frac{1}{6}-\frac{4}{15}$
- 6=2*3
- 15=3*5 so the LCD is 2*3*5=30

$$\frac{1}{6}-\frac{4}{15}=\frac{1}{2\cdot 3}-\frac{4}{3\cdot 5}=\frac{1\cdot 5}{2\cdot 3\cdot 5}-\frac{4\cdot 2}{3\cdot 5\cdot 2}=\frac{5}{30}-\frac{8}{30}=\frac{-3}{30}=\frac{-1\cdot 3}{10\cdot 3}=-\frac{1}{10}$$

** Ex. 13-22**

Now adding polynomials again… you gotta love 'em! Ex3 pg 405

- a) $$\frac{2}{3y}+\frac{4}{3y}=\frac{6}{3y}=\frac{2}{y}$$

- b) $$\frac{2x}{x+2}+\frac{4}{x+2}=\frac{2x+4}{x+2}=\frac{2(x+2)}{x+2}=2$$

- c) next page

Ex 3b

$$\frac{x^2+2x}{(x-1)(x+3)}-\frac{2x+1}{(x-1)(x+3)}=\frac{x^2+2x-(2x+1)}{(x-1)(x+3)}=$$

$$\frac{x^2+2x-2x-1}{(x-1)(x+3)}=\frac{x^2-1}{(x-1)(x+3)}=\frac{(x-1)(x+1)}{(x-1)(x+3)}=\frac{x+1}{x+3}$$

** Ex. 23-34**

Now we mix up the denominators (they won't match… so we must make them!) Ex 4

- a) $$\frac{5}{2x}+\frac{2}{3}$$
- The LCD is 2x*3 = 6x

$$\frac{5}{2x}+\frac{2}{3}=\frac{5\cdot 3}{2x\cdot 3}+\frac{2\cdot 2x}{3\cdot 2x}=\frac{15}{6x}+\frac{4x}{6x}=\frac{15+4x}{6x}$$

Now 4b

- b) $\frac{4}{x^3 y} + \frac{2}{xy^3}$

- x^3y
- xy^3 So the LCD is x^3y^3

$$\frac{4}{x^3 y} + \frac{2}{xy^3} = \frac{4 \cdot y^2}{x^3 y \cdot y^2} + \frac{2 \cdot x^2}{xy^3 \cdot x^2} = \frac{4y^2}{x^3 y^3} + \frac{2x^2}{x^3 y^3} = \frac{4y^2 + 2x^2}{x^3 y^3}$$

Ex 4c

- b) $$\frac{a+1}{6}-\frac{a-2}{8}$$

- 6=2*3
- 8=2^3 So the LCD is 2^3 *3 = 24

$$\frac{a+1}{6}-\frac{a-2}{8}=\frac{(a+1)4}{6\cdot 4}-\frac{(a-2)3}{8\cdot 3}=\frac{4a+4}{24}-\frac{3a-6}{24}=$$

$$\frac{4a+4-(3a-6)}{24}=\frac{4a+4-3a+6}{24}=\frac{a+10}{24}$$

** Ex. 35-50**

ok, different denominators, and they are polynomials Ex 5a pg 407

- a) $$\frac{1}{x^2-9}+\frac{2}{x^2+3x}$$

x^2-9= (x-3)(x+3) needs x

x^2+3x= x(x+3) needs (x-3)

$$\frac{1}{x^2-9}+\frac{2}{x^2+3x}=\frac{1\cdot x}{(x-3)(x+3)x}+\frac{2(x-3)}{(x-3)(x+3)x}=$$

$$\frac{x+2x-6}{x(x-3)(x+3)}=\frac{3x-6}{x(x-3)(x+3)}$$

Ex 5b

- b)

$$\frac{4}{5-a}-\frac{2}{a-5}=\frac{4(-1)}{(5-a)(-1)}-\frac{2}{a-5}=$$

$$\frac{-4}{a-5}-\frac{2}{a-5}=\frac{-6}{a-5}$$

** Ex. 51-68**

Now for a triple, rolling, double axle, with a twist.
Ex 6 page 407

- Buckle your seatbelts, it's as bad as it gets.

$$\frac{x+1}{x^2+2x}+\frac{2x+1}{6x+12}-\frac{1}{6}$$

$$\frac{x+1}{x^2+2x}+\frac{2x+1}{6x+12}-\frac{1}{6}=$$

$$\frac{x+1}{x(x+2)}+\frac{2x+1}{6(x+2)}-\frac{1}{6}=$$

$$\frac{6(x+1)}{6x(x+2)}+\frac{x(2x+1)}{6x(x+2)}-\frac{1x(x+2)}{6x(x+2)}=$$

$$\frac{6x+6}{6x(x+2)}+\frac{2x^2+x}{6x(x+2)}-\frac{x^2+2x}{6x(x+2)}=$$

$$\frac{6x+6+2x^2+x-x^2-2x}{6x(x+2)}=\frac{x^2+5x+6}{6x(x+2)}=$$

$$\frac{(x+3)(x+2)}{6x(x+2)}=\frac{x+3}{6x}$$

** Ex. 69-74**

Now try your hand, until it falls off… Section 6.4

- Definitions Q1-4
- Just numbers/fractions Q5-12
- More numbers reduced Q13-22
- Now add monomials Q23-34
- Different denominators monomials Q35-50
- Handle what comes along! Q51-84
- Word probs Q85-92

Section 6.5 Complex Fractions

Numerator of the complex fraction

$$\frac{\frac{1}{2}+\frac{2}{3}}{\frac{1}{4}-\frac{5}{8}}$$

Denominator of the complex fraction

Example 1 page 413
Simplifying complex fractions

a) $$\frac{\frac{1}{2}+\frac{2}{3}}{\frac{1}{4}-\frac{5}{8}}$$

Numerator first: $$\frac{1}{2}+\frac{2}{3}=\frac{1\cdot 3}{2\cdot 3}+\frac{2\cdot 2}{3\cdot 2}=\frac{3}{6}+\frac{4}{6}=\frac{7}{6}$$

Denominator second:

$$\frac{1}{4}-\frac{5}{8}=\frac{1\cdot 2}{4\cdot 2}-\frac{5}{8}=\frac{2}{8}-\frac{5}{8}=-\frac{3}{8}$$

More of ex.1a page 413

$$\frac{\frac{1}{2}+\frac{2}{3}}{\frac{1}{4}-\frac{5}{8}}=\frac{\frac{7}{6}}{-\frac{3}{8}}=\frac{7}{6}\div\left(-\frac{8}{3}\right)=\frac{7}{6}\cdot\left(-\frac{3}{8}\right)=-\frac{56}{18}=-\frac{28}{9}$$

b)

$$\frac{4-\frac{2}{5}}{\frac{1}{10}+3}=\frac{\frac{20}{5}-\frac{2}{5}}{\frac{1}{10}+\frac{30}{10}}=\frac{\frac{18}{5}}{\frac{31}{10}}=\frac{18}{5}\div\frac{31}{10}=\frac{18}{5}\cdot\frac{10}{31}=\frac{18}{1}\cdot\frac{2}{31}=\frac{36}{31}$$

** Ex. 3-14**

LCD Strategies

1. Find the LCD for all the denominators in the complex fraction
2. Multiply both the numerator and the denominator of the complex fraction by the LCD. Use the distributive property if necessary.
3. Combine like terms if possible.
4. Reduce to lowest terms when possible.

Example 2 page 414– Using LCD to simplify

$$\frac{\frac{1}{2}+\frac{2}{3}}{\frac{1}{4}-\frac{5}{8}}=\frac{\left(\frac{1}{2}+\frac{2}{3}\right)\cdot 24}{\left(\frac{1}{4}-\frac{5}{8}\right)\cdot 24}=\frac{\frac{1}{2}\cdot 24+\frac{2}{3}\cdot 24}{\frac{1}{4}\cdot 24-\frac{5}{8}\cdot 24}=\frac{12+16}{6-15}=\frac{28}{-9}=-\frac{28}{9}$$

** Ex. 15-22**

Example 3 page 415
Doing with some x's inside

$$\frac{2-\frac{1}{x}}{\frac{1}{x^2}-\frac{1}{2}}=\frac{\left(2-\frac{1}{x}\right)\left(2x^2\right)}{\left(\frac{1}{x^2}-\frac{1}{2}\right)\left(2x^2\right)}=\frac{2\cdot 2x^2-\frac{1}{x}\cdot 2x^2}{\frac{1}{x^2}\cdot 2x^2-\frac{1}{2}\cdot 2x^2}=\frac{4x^2-2x}{2-x^2}$$

** Ex 23-32**

Example 4 page 415
Another example

$$\frac{\frac{1}{x-2}-\frac{2}{x+2}}{\frac{3}{2-x}+\frac{4}{x+2}}=\frac{\frac{1}{x-2}(x-2)(x+2)-\frac{2}{x+2}(x-2)(x+2)}{\frac{3}{2-x}(x-2)(x+2)+\frac{4}{x+2}(x-2)(x+2)}=$$

$$=\frac{x+2-2(x-2)}{3\left(\frac{x-2}{2-x}\right)(x+2)+4(x-2)}=\frac{x+2-2(x-2)}{3(-1)(x+2)+4(x-2)}=\frac{x+2-2x+4}{-3x-6+4x-8}=\frac{-x+6}{x-14}$$

** Ex 33-48**

Pencil scratching time
Section 6.5

- Definitions Q1-3
- Complex Fractions Q4-14
- Using the LCD to simplify Q15-62
- Applications Q63-66

Now we jump to section 6.6

- And we change gears to solve equations with rational (ratios or fractions) in them.
- Here they are putting the variable down in the bottom of the fraction.
- Yucky? Well, not if you go step by step!

We last saw this in 2.6, now x goes downstairs.

- But first, a review… x in the attic.

Example 1
page 420

** Ex. 5-16**

$$Solve\ \frac{1}{2}-\frac{x-2}{3}=\frac{1}{6},$$

$$6\left(\frac{1}{2}-\frac{x-2}{3}\right)=6\cdot\frac{1}{6},$$

$$6\cdot\frac{1}{2}-6\cdot\frac{x-2}{3}=6\cdot\frac{1}{6},$$

$$3-2(x-2)=1,$$

$$3-2x+4=1,$$

$$-2x=-6$$

$$x=3$$

Example 2 pg 421
NOW we put x in the basement

$$\frac{1}{x}+\frac{1}{6}=\frac{1}{4},$$

$$12x\left(\frac{1}{x}+\frac{1}{6}\right)=12x\left(\frac{1}{4}\right),$$

$$12x\cdot\frac{1}{x}+12x\cdot\frac{1}{6}=12x\cdot\frac{1}{4},$$

$$12+2x=3x$$

$$12=x$$

** Ex. 17-28**

Example 3 pg 421 One with two solutions…

$$\frac{100}{x}+\frac{100}{x+5}=9,$$

$$x(x+5)\frac{100}{x}+x(x+5)\frac{100}{x+5}=x(x+5)9,$$

$$x(x+5)\frac{100}{x}+x(x+5)\frac{100}{x+5}=x(x+5)9,$$

$$(x+5)100+x(100)=(x^2+5x)9,$$

$$100x+500+100x=9x^2+45x,$$

$$500+200x=9x^2+45x,$$

$$0=9x^2-155x-500,$$

$$0=(9x+25)(x-20),$$

$$so...9x+25=0\_or\_x-20=0$$

$$giving...\_x=-\frac{25}{9}or\_x=20$$

One denominator is x the other is x+5, so the LCD is x(x+5)

** Ex. 29-36**

Exploding Equations Batman!

- Extraneous Solutions: We haven't done it every time in the power point presentations, but you need to plug the numbers back into the original equations IF there is a variable in the DENOMINATOR (bottom of the fractions).
- It MIGHT = 0 so you have stuff/0 = BAD!
- These are called *Extraneous Solutions*

Example 4 pg 422 Extraneous Ans.

- One denominator is (x-2) the other is 2(x-2) so the LCD is 2(x-2)

** Ex. 37-40**

$$\frac{1}{x-2}=\frac{x}{2x-4}+1,$$

$$2(x-2)\frac{1}{x-2}=2(x-2)\frac{x}{2(x-2)}+2(x-2)\cdot 1,$$

$$2(x-2)\frac{1}{x-2}=2(x-2)\frac{x}{2(x-2)}+2(x-2)\cdot 1,$$

$$2=x+2x-4,$$

$$2=3x-4,$$

$$6=3x,$$

$$2=x,$$

$$plug\_it\_in,$$

$$\frac{1}{2-2}=\frac{2}{2\cdot 2-4}+1$$

Another explosive one…

Ex5 pg423

- One denominator is x the other two are x-3, so the LCD is x(x-3)

$$\frac{1}{x}+\frac{1}{x-3}=\frac{x-2}{x-3},$$

$$x(x-3)\cdot\frac{1}{x}+x(x-3)\cdot\frac{1}{x-3}=x(x-3)\cdot\frac{x-2}{x-3},$$

$$x(x-3)\cdot\frac{1}{x}+x(x-3)\cdot\frac{1}{x-3}=x(x-3)\cdot\frac{x-2}{x-3},$$

$$x-3+x=x(x-2),$$

$$2x-3=x^2-2x,$$

$$0=x^2-4x+3,$$

$$0=(x-3)(x-1),$$

$$x-3=0\_or\_x-1=0,$$

$$so\_x=3\_or\_x=1,$$

$$plug\_it\_in,$$

$$\frac{1}{3}+\frac{1}{3-3}=\frac{1}{3-3}$$

3= explosion

1 = a good solution, and the only one

** Ex. 41-44**

Make sure you check!

Always check those answers, they MAY explode, or you may have made a math error.

Section 6.6 Being Solvent…

- Definitions Q1-4
- Solve equations with x on top Q5-16
- Solve the equations x on bottom Q17-38
- Solve watching for extraneous solutions Q39-44
- Solve each Q45-Q58
- Word problems Q59-68

Section 6.7 What were those Ratios all about?

- Now we do some application (a breather in the midst of the Algebra Blizzard).

Ratios

- Way back in Chapter 1 we defined a rational number as the ratio of two integers (is that on your white index cards?).
- Now we'll go a step further…
- If a and b are any real number (not just integers) and b isn't 0, then a/b is called the ratio of a and b. OR the ratio of a to b.

Compare Compare Compare

- A ratio is just the comparison of one number to the other.
- You do this instinctively in your day to day life.

A picture book of the critters

$$\frac{3}{4}, \frac{4.2}{2.1}, \frac{\frac{1}{4}}{\frac{1}{2}}, \frac{3.6}{5}, \frac{100}{1}$$

Finding equivalent ratios

- Find an equivalent ratio integers in the lowest terms for each ratio
- a) $$\frac{4.2}{2.1} = \frac{4.2(10)}{2.1(10)} = \frac{42}{21} = \frac{21 \cdot 2}{21 \cdot 1} = \frac{2}{1}$$

We're working with ratios so leave the 1 in the denominator! (Go ahead, be lazy.)

Ex 1b

- b)

$$\frac{\frac{1}{4}}{\frac{1}{2}} = \frac{\frac{1}{4}\cdot 4}{\frac{1}{2}\cdot 4} = \frac{1}{2}$$

** Ex. 7-22**

- c)

$$\frac{3.6}{5} = \frac{3.6\cdot 10}{5\cdot 10} = \frac{36}{50} = \frac{18}{25}$$

Enter Stage Left, the *Word Problems*

- (Who made this a horror show?)

- Ratios lie at the root of many day to day problems…

Example 2 page 426-7

- In a 50lb bag of lawn fertilizer, there are 8 pounds of nitrogen and 12 pounds of potash. What is the ratio of nitrogen to potash?

$$\frac{8}{12} = \frac{2 \cdot 4}{3 \cdot 4} = \frac{2}{3}$$

- So the ratio of nitrogen to potash is 2 to 3 or 2:3

** Ex. 23-24**

Example 3 page 427

- In a class of 50 students, there were exactly 20 male students. What was the ratio of males to females in class?

- Because there are 20 male students, there must be 30 female students. The ratio of males to females is 20/30, or 2 to 3 (or 2:3)

** Ex. 25-26**

Example 4 page 427

- What is the ratio of length to width for a poster with a length of 30 inches and a width of 2 feet?
- Note, 2 feet is 24 inches. So the ratio is 30 to 24.
- $$\frac{30}{24} = \frac{5 \cdot 6}{4 \cdot 6} = \frac{5}{4}$$
- and the ratio length to width is 5 to 4.

** Ex. 27-30**

Proportions

- It is any statement expressing the equality of two ratios. It can be expressed in either notation:

$$\frac{a}{b} = \frac{c}{d} \quad or \quad a : b = c : d$$

More ratio definitions

$$\frac{a}{b} = \frac{c}{d} \quad or \quad a:b = c:d$$

- a and d are called extremes
- c and b are called the means

- a*d=c*d   or
- 30*4=5*24 Cool! No?

$$\frac{30}{24} = \frac{5}{4}$$

LCD and ratios

- Multiply by the LCD bd you get…

$$\frac{a}{b} \cdot bd = \frac{c}{d} \cdot bd$$

$$ad = bc$$

Extremes-Means Property
(cross multiplying)

$$\frac{a}{b} = \frac{c}{d}, then \quad ad = bc$$

Example 5
Secrets of the extremes-means property
page 428

$$\frac{3}{x} = \frac{5}{5+x},$$

$$3(x+5) = 5x,$$

$$3x+15 = 5x,$$

$$15 = 2x,$$

$$\frac{15}{2} = x$$

** Ex. 31-44**

Example 6 page 429

- Let x be the number of catfish in pond. The ratio 30/x is the ratio of tagged catfish to the total population. The ratio of 3/500 is the ratio of tagged catfish in the sample to the sample size. If catfish are really well mixed and the sample is random, the ratios should be equal.

Ex 6 continued

$$\frac{30}{x} = \frac{3}{500},$$

$$3x = 15,000,$$

$$x = 5000$$

- So there are 5000 catfish in the pond.

** Ex. 45-48**

Example 7 page 429 now for a proportion

- In a conservative portfolio the ratio of the amount invested in bonds to the amount invested in stocks should be 3 to 1 (or 3:1). A conservative investor invested $2850 more in bonds than she did in stocks. How much did she invest in each category?

Ex 7 now for the answer…..

$$\frac{Amount\_invested\_in\_bonds}{Amount\_invested\_in\_stocks} = \frac{3}{1},$$

$$\frac{x+2850}{x} = \frac{3}{1},$$

$$3x = x + 2850,$$

$$2x = 2850,$$

$$x = 1425,$$

$$that\_x + 2850 = 4275$$

** Ex. 49-52**

- So she invested $4275 in bonds and $1425 in stocks

Example 8 page 430

- There are 3 feet in 1 yard. How many feet are there in 12 yards?

$$\frac{3\,feet}{x feet} = \frac{1 yard}{12\,yards}, or, \frac{3\,feet}{1 yard} = \frac{x feet}{12\,yards}$$

- So you get 3*12=x*1 or x=36
- Which means there are 36feet in 12 yards

** Ex. 53-56**

And more… Section 6.7

- Definitions Q1-6
- Ratios Q7-22
- Applications Q23-30
- Proportions Q31-44
- Applications proportions Q45-67

Section 6.8 Applications Appli-smations

- Now we link much of what you've seen earlier together with the ideas of the RATIO.

Example 1 page 434
Equation of a line

- If you are given the point and slope that defines a line (using the point-slope form) of (-2,4) and 3/2 given:
- You could go,
- $y-y_1=m(x-x_1)$
- y-4=3/2(x+2)
- etc… OR

$$\frac{y-4}{x+2}=\frac{3}{2},$$

$$(x+2)\cdot\frac{y-4}{x+2}=(x+2)\cdot\frac{3}{2},$$

$$y-4=\frac{3}{2}x+3,$$

$$y=\frac{3}{2}x+7$$

** Ex. 1-10**

Example 2 page 434
Distance, rate, time…

- Solve the formula

$$\frac{D}{T} = R\_for\_T$$

** Ex. 11-16**

$$\frac{D}{T} = R,$$

$$T \cdot \frac{D}{T} = T \cdot R,$$

$$D = TR$$

$$\frac{D}{R} = \frac{TR}{R},$$

$$\frac{D}{R} = T$$

Example 3 page 435 the setup

- The formula $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ gives the relationship between the resistances in a circuit. Solve the formula for R_2 .

Example 3 the solution…

- The LCD between R, R_1, R_2 *is* RR_1R_2

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$RR_1R_2 \cdot \frac{1}{R} = RR_1R_2 \cdot \frac{1}{R_1} + RR_1R_2 \cdot \frac{1}{R_2}$$

$$R_1R_2 = RR_2 + RR_1$$

$$R_1R_2 - RR_2 = RR_1$$

$$R_2(R_1 - R) = RR_1$$

$$R_2 = \frac{RR_1}{R_1 - R}$$

** Ex. 17-24**

Example 4 pg 435 The value of a variable

- In the formula

in Ex 1,

find x if y=-3

$$\frac{y-4}{x+2}=\frac{3}{2},$$

$$\frac{-3-4}{x+2}=\frac{3}{2},$$

$$\frac{-7}{x+2}=\frac{3}{2},$$

$$3x+6=-14,$$

$$3x=-20,$$

$$x=-\frac{20}{3}$$

** Ex. 24-34**

What's helpful with motion (distance,time,rate) problems

- Remember:

D=RT gives us distances

$T = \frac{D}{R},$ gives us times (and looks like a ratio)

$$\frac{T}{1} = \frac{D}{R}$$

Example 5 page 436
Thinking of Beaches

- Susan drove 1500 miles to Daytona Beach for spring break. On the way back she averaged 10 miles per hour less, and the drive back took her 5 hours longer. Find Susan's average speed on the way to Daytona Beach
- We'll say her average speed (going there) is x. Then x-10 is here average speed coming home.
- We'll use $T = \frac{D}{R}$ to make the table…

Ex 5 solving

	D	**R**	**T**
Going to the beach	1500	x	1500/x
Returning from the beach	1500	x-10	1500/(x-10)

More Ex 5

- We know that:

longer time – shorter time =5

So we'll take the longer time from the table and subtract the shorter time from the table and make it equal 5

Example 5 Big fat equations

$$\frac{1500}{x-10}-\frac{1500}{x}=5,$$

$$x(x-10)\frac{1500}{x-10}-x(x-10)\frac{1500}{x}=x(x-10)5,$$

$$1500x-1500(x-10)=5x^2-50x,$$

$$15000=x^2-10x,$$

$$3000=x^2-10x,$$

$$0=x^2-10x-3000,$$

$$=(x+50)(x-60)=0,$$

$$x+50=0 \quad or \quad x-60=0$$

$$so \quad x=-50 \quad or \quad x=60$$

Wrapping Ex 5

- -50mph is dumb, so here average speed is the positive answer = 60mph going out to the beach.

** Ex. 35-40**

Example 6 pg 436-7

- After a heavy snowfall, Brian can shovel all the driveway in 30 minutes. If his younger brother Allen helps, the job takes only 20 minutes. How long would it take Allen to do the job by himself?

Example 6

- x will be the number of minutes it would take Allen to do the job by himself. Brian’s rate for shoveling is 1/30 of the driveway per minute, and Allen’s rate for shoveling is 1/x of the driveway per minute. We organize all of the information in a table like the table in Ex 5

Example 6 tabling the motion

	RATE	TIME	WORK
Brian	1/30 job/min.	20 min.	=2/3 job
Allen	1/x job/min.	20 min.	=20/x job

Equating it…

- If you add their work, they do the whole job (or 1 snow shoveling job)

$$\frac{2}{3}+\frac{20}{x}=1,$$

$$3x\cdot\frac{2}{3}+3x\cdot\frac{20}{x}=3x\cdot 1,$$

$$2x+60=3x,$$

$$60=x$$

Cleaning up Ex 6

- So if it takes Allen 60 minutes to do the job himself, then he must be working at the rate of 1/60th of the job per minute. In 20 minutes he does 1/3rd of the job while Brian does 2/3^{rds} of the job.
- So it will take Allen 60 minutes to do it by himself.

** Ex. 41-46**

Helps for Solving "Work" Problems

1. If a job is completed in x hours, then the rate is
2. Make a table showing rate, time and work completed (W=R*T) for each person or machine is $\frac{1}{x} \frac{job}{hours}$
3. The total work completed is the sum of the individual amounts of work completed
4. If the job is completed, then the total work done is 1 job.

Purchasing Probs.

- A neat way to look at it… rates!
- If your gas is 1.74 cents/gallon, that is the rate at which your bill is increasing as you pump the gas in the tank.
- The product of the rate and the quantity purchased is the total cost.

Example 8 page 438

- Oranges…and Grapefruit?
- Tamara bought 50 lbs of fruit consisting of both Florida Oranges and Texas Grapefruits. She paid twice as much per pound for grapefruit as she did for oranges. If she bought $12 worth of oranges and $16 worth of grapefruit, how many pounds of each did she buy?
- x = the number of lbs of Oranges, and 5-x the pounds of Grapefruit

Ex 8 the Table

	RATE	**Quantity**	**Total Cost**
Oranges	12/x dollars/lb	x pounds	12 dollars
Grapefruit	$\frac{16}{50-x}\frac{dollars}{lb}$	50-x lbs	16 dollars

Example 8 the equation

- Since the price per pound for the grapefruit is twice that for the oranges, we have:
- 2(price per pound Oranges)=price per pound Grapefruit

$$2\left(\frac{12}{x}\right)=\frac{16}{50-x},$$

$$\frac{24}{x}=\frac{16}{50-x},$$

$$16x=1200-24x,$$

$$40x=1200,$$

$$x=30,$$

$$50-x=20$$

Example 8 cleaning up

- So she bought 20 pounds of grapefruit (for $16 this is $0.80 per pound).
- And she bought 30 pounds of Oranges (for $12 which is $0.40 per pound).
- Note the price of grapefruit is 2* the price of the oranges as expected!

** Ex. 49-50**

Blaa Blaa Blaa Practice Section 6.8

- Solving for y Q 1-10
- Solve for what you are asked to solve for
  Q11-24
- Find the value (plug in numbers) Q25-34
- Word problems Q34-64