University of Phoenix
MTH 209 Algebra II

Week 4

The FUN FUN FUN continues!

A jump to Radicals! Section 9.1

- What is a radical?

- If you have $2^2=4$ Then 2 is the root of four (or square root)
- If you have $2^3=8$ Then 2 is the cube root of eight.

Definitions

- nth Roots

- If $a=b^n$ for a positive integer n, then :

b is the nth root of a

Specifically if $a = b^2$ then b is the square root

And if $a=b^3$ then b is the cube root

More definitions

- If n is even then you have *even roots*.

The positive even root of a positive even number is called the *principle root*

For example, the principle square root of 9 is 3 The principle fourth root of 16 is 2

- If n is odd, then you find *odd roots*.

Because $2^5 = 32$, 2 is the fifth root of 32.

Enter…the…

$$\sqrt{\quad}$$

The Radical Symbol

- We use the $\sqrt{}$ symbol for a root
- So we can define : $\sqrt[n]{a}$

where if **n** is positive and even and a is positive, then that denotes the principle nth root of **a**

If **n** is a positive odd integer, then it denotes the nth root of **a**

If n is any positive integer then $\sqrt[n]{0} = 0$

The radical *radicand*

- We READ $\sqrt[n]{a}$ as “the *n*th root of a”
- The n is the *index of the radical*
- The a is the *radicand*

Example 1 pg 542

- Find the following roots
- a) $\sqrt{25}$ because $5^2 = 25$, the answer is 5
- b) $\sqrt[3]{-27}$ because $(-3)^{3} = -27$ the answer is –3
- c) $\sqrt[6]{64}$ because $2^6 = 64$ the answer is 2
- d) $-\sqrt{4}$ because $\sqrt{4} = 2$, $-\sqrt{4} = -2$

** Ex. 7-22**

A look into horror!

- What's up with these?

$$\sqrt{-9} \qquad \sqrt[4]{-81} \qquad \sqrt[6]{-64}$$

- What two numbers (or 4 or 6) satisfy these?

- They are *imaginary* they are NOT real numbers and will be NOT be dealt with in section 9.6 – it was removed in this version of the class.

Roots and Variables

- Definition: *Perfect Squares*

x^2 ,x^4 ,x^6 ,x^8 ,x^{10}, x^{12} ,…

EASY to deal with

$$\sqrt{x^2} = x \qquad \sqrt{x^4} = x^2 \qquad \sqrt{x^6} = x^3$$

Cube roots and Variables

- Definition: *Perfect Cubes*

x^3 ,x^6 ,x^9 ,x^{12} ,x^{15}, x^{18} ,…

EASY to deal with with CUBE roots

$$\sqrt[3]{x^3} = x \qquad \sqrt[3]{x^6} = x^2 \qquad \sqrt[3]{x^9} = x^3$$

Example 2 – Roots of exponents page 544

- a) $\sqrt{x^{22}} = x^{11}$
- b) $\sqrt[3]{t^{18}} = t^{6}$
- c) $\sqrt[5]{s^{30}} = s^{6}$

** Ex. 23-34**

The totally RADICAL product rule for radicals

- What if you have $\sqrt{2}\cdot\sqrt{3}$ and you want to square it?

$$(\sqrt{2}\cdot\sqrt{3})^2 = (\sqrt{2})^2\cdot(\sqrt{3})^2 = 2\cdot 3 = 6$$

The Product Rule for Radicals

- The *n*th root of a product is equal to the product of the *n*th roots. Which looks like:

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

provided all of these roots are real numbers.

Example 3 Using said product rule page 545

- a) $\sqrt{4y} = \sqrt{4} \cdot \sqrt{y} = 2 \cdot \sqrt{y} = 2\sqrt{y}$

- b) $\sqrt{3y^8} = \sqrt{3} \cdot \sqrt{y^8} = \sqrt{3} \cdot y^4 = y^4\sqrt{3}$

By a convention of old people, we normally put the radical on the right…

Ex 3c

c)

$$\sqrt[3]{125w^2} = \sqrt[3]{125} \cdot \sqrt[3]{w^2} = 5\sqrt[3]{w^2}$$

** Ex. 35-46**

Example 4 pg 545

a) $\sqrt{12} = \sqrt{4 \cdot 3} = \sqrt{4} \cdot \sqrt{3} = 2\sqrt{3}$

b) $\sqrt[3]{54} = \sqrt[3]{27 \cdot 2} = \sqrt[3]{27} \cdot \sqrt[3]{2} = 3\sqrt[3]{2}$

c) $\sqrt[4]{80} = \sqrt[4]{16 \cdot 5} = \sqrt[4]{16} \cdot \sqrt[4]{5} = 2\sqrt[4]{5}$

d) $\sqrt[5]{64} = \sqrt[5]{32 \cdot 2} = \sqrt[5]{32} \cdot \sqrt[5]{2} = 2\sqrt[5]{2}$

** Ex 47-60**

Example 5 pg 546

a) $\sqrt{20x^3} = \sqrt{4x^2 \cdot 5x} = \sqrt{4x^2} \cdot \sqrt{5x} = 2x\sqrt{5x}$

b) $\sqrt[3]{40a^8} = \sqrt[3]{8a^6 \cdot 5a^2} = \sqrt[3]{8a^6} \cdot \sqrt[3]{5a^2} = 2a^2\sqrt[3]{5a^2}$

c) $\sqrt[4]{48a^4b^{11}} = \sqrt[4]{48a^4 \cdot b^{11}} = \sqrt[4]{48a^4} \cdot \sqrt[4]{b^{11}} = 2ab^2\sqrt[4]{3b^3}$

d) $\sqrt[5]{w^7} = \sqrt[5]{w^5 \cdot w^2} = \sqrt[5]{w^5} \cdot \sqrt[5]{w^2} = w\sqrt[5]{w^2}$

** Ex 61-74**

What about those Quotients?

- The *n*th root of a quotient is equal to the quotient of the *n*th roots. In symbols it looks like:

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

- Remember, b can't equal zero! And all of these roots must be real numbers.

Example 6 pg 547
Simplify radically

a) $\sqrt{\frac{25}{9}} = \frac{\sqrt{25}}{\sqrt{9}} = \frac{5}{3}$

b) $\frac{\sqrt{15}}{\sqrt{3}} = \sqrt{\frac{15}{3}} = \sqrt{3}$

c) $\sqrt[3]{\frac{b}{125}} = \frac{\sqrt[3]{b}}{\sqrt[3]{125}} = \frac{\sqrt[3]{b}}{5}$

d) $\sqrt[3]{\frac{x^{21}}{y^{6}}} = \frac{\sqrt[3]{x^{21}}}{\sqrt[3]{y^{6}}} = \frac{x^{7}}{y^{2}}$

** Ex. 75-86 **

Example 7 pg 547

Simplify radically with prod. & quot. rule

a)

$$\sqrt{\frac{50}{49}} = \frac{\sqrt{25}\cdot\sqrt{2}}{\sqrt{49}} = \frac{5\sqrt{2}}{7}$$

b)

$$\sqrt[3]{\frac{x^5}{8}} = \frac{\sqrt[3]{x^5}\cdot\sqrt[3]{x^2}}{\sqrt[3]{8}} = \frac{x\sqrt[3]{x^2}}{2}$$

c)

$$\sqrt[4]{\frac{a^5}{b^8}} = \frac{\sqrt[4]{a^4}\cdot\sqrt[4]{a}}{\sqrt[4]{b^8}} = \frac{a\sqrt[4]{a}}{b^2}$$

** Ex. 75-86 **

Example 8 page 548
Domani'

a) $\sqrt{x-5}$ You just don't want the $\sqrt{negative\_number}$

So $x-5 \geq 0 ... x \geq 5$

b) $\sqrt[3]{x+7}$ So all x's are ok!

c) $\sqrt[4]{2x+6}$ This time all $x \geq -3$ are allowed

Section 9.1 Riding the Radical

- Definitions Q1-Q6
- Find the root in numbers Q7-22
- Find the root in variables Q23-Q34
- Use the product rule to simplify Q35-Q74
- Quotient rule Q75-98
- The domain Q99-106
- Word problems Q107-117

9.2 Rati0nal Exp0nents

- This is the rest of the story!

- Remember how we looked at the *spectrum* of powers?

$2^3=8$

$2^2=4$

$2^1=2$

$2^0=1$

$2^{-1}=1/2$

$2^{-2}=1/4$

Defining it…

- If n is any positive integer then…

$$a^{1/n} = \sqrt[n]{a}$$

- Provided that $\sqrt[n]{a}$ is a real number

9.2 Ex. 1 pg 553
More-on Quadratic Equations

a) $\sqrt[3]{35} = 35^{1/3}$

b) $\sqrt[4]{xy} = (xy)^{1/4}$

c) $5^{1/2} = \sqrt{5}$

d) $a^{1/3} = \sqrt[3]{a}$

Ex. 7-14

Example 2 pg 55

Finding the roots **Ex. 15-22**

a) $4^{1/2} = \sqrt{4} = 2$

b) $(-8)^{1/3} = \sqrt[3]{-8} = -2$

c) $81^{1/4} = \sqrt[4]{81} = 3$

d) $(-9)^{1/2} = \sqrt{-9} not\_real$

e) $-9^{1/2} = -\sqrt{9} = -3$

The exponent can be anything!

The numerator is the power

$$\frac{2}{3}$$

The denominator is root

Another definition…

$$a^{m/n} = (a^{1/n})^m = (a^m)^{1/n}$$

or

$$a^{m/n} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$$

And negatives?

Just upside down.

$$a^{-m/n} = \frac{1}{a^{m/n}}$$

or

$$a^{-m/n} = \frac{1}{(\sqrt[n]{a})^m}$$

Example 3 pg 554
Changing radicals to exponents

a) $\sqrt[3]{a^2} = x^{2/3}$

b) $\dfrac{1}{\sqrt[4]{m^3}} = \dfrac{1}{m^{3/4}} = m^{-3/4}$

** Ex. 23-26**

Example 4 page 554
Exponents going to radicals

a) $5^{2/3} = \sqrt[3]{5^2}$

b) $a^{-2/5} = \dfrac{1}{\sqrt[5]{a^2}}$

Ex. 27-30

Cookbook 1

$$a^{-m/n}$$

Reciprocal Power Root

Cookbook 2

1. Find the nth root of a
2. Raise your result to the nth power
3. Find the reciprocal

Example 5 page 555
Rational Expressions

a) $$27^{2/3} = (27^{1/3})^2 = 3^2 = 9$$

b) $$4^{-3/2} = \frac{1}{(4^{1/2})^3} = \frac{1}{2^3} = \frac{1}{8}$$

c) $$81^{-3/4} = \frac{1}{(81^{1/4})^3} = \frac{1}{3^3} = \frac{1}{27}$$

d) $$(-8)^{-5/3} = \frac{1}{((-8)^{1/3})^5} = \frac{1}{(-2)^5} = \frac{1}{-32} = -\frac{1}{32}$$

Ex. 31-42

Everything at a glance (remember these from 2x before?)

Rules for Rational Exponents

The following rules hold for any nonzero real numbers a and b and rational numbers r and s for which the expressions represent real numbers.

1. $a^r a^s = a^{r+s}$ Product rule
2. $\frac{a^r}{a^s} = a^{r-s}$ Quotient rule
3. $(a^r)^s = a^{rs}$ Power of a power rule
4. $(ab)^r = a^r b^r$ Power of a product rule
5. $\left(\frac{a}{b}\right)^r = \frac{a^r}{b^r}$ Power of a quotient rule

Example 6 pg 556
Using product and quotient rules

a) $$27^{1/6} \cdot 27^{1/2} = 27^{1/6+1/2} = 27^{2/3} = 9$$

b) $$\frac{5^{3/4}}{5^{1/4}} = 5^{3/4-1/4} = 5^{2/4} = 5^{1/2} = \sqrt{5}$$

Ex 43-50

Example 7 pg 557

Power to the Exponents! **Ex.51-60**

a) $3^{1/2} \cdot 12^{1/2} = (3 \cdot 12)^{1/2} = 36^{1/2} = 6$

b) $(3^{10})^{1/2} = 3^5$

c) $\left(\dfrac{2^6}{3^9}\right) = \dfrac{\left(2^6\right)^{-1/3}}{\left(3^9\right)^{-1/3}} = \dfrac{2^{-2}}{3^{-3}} = \dfrac{3^3}{2^2} = \dfrac{27}{4}$

Square roots have 2 answers!

$$(x^2)^{1/2} = |x|$$

$$(same\_as)$$

$$\sqrt{x^2} = |x|$$

Ex 8 pg 558 – So you use absolute values with roots…

a) $(x^8y^4)^{1/4} = x^2|y|\_or\_|x^2y|$

b) $\left(\frac{x^9}{8}\right)^{1/3} = \frac{x^3}{2}$

Ex. 61-70

Ex 9 pg 558 Mixed Bag **Ex. 71-82**

a) $x^{2/3}y^{4/3} = x^{6/3} = x^2$

b) $\dfrac{a^{1/2}}{a^{1/4}} = a^{1/2-1/4} = a^{1/4}$

c) $(x^{1/2}y^{-3}) = (x^{1/2})^{1/2}(y^{-3})^{1/2} = x^{1/4}y^{-3/2} = \dfrac{x^{1/4}}{y^{3/2}}$

d) $\left(\dfrac{x^2}{y^{1/3}}\right)^{-1/2} = \left(\dfrac{y^{1/3}}{x^2}\right)^{1/2} = \dfrac{y^{1/6}}{x}$

Section 9.2 Radical Ideas

- Definitions Q1-Q6
- Rational Exponents Q7-42
- Using the rules of Exponents Q43-Q60
- Simplifying things with letters Q61-Q82
- Mixed Bag Q83-126
- Word problems Q127-136

Section 9.3 Now adding, subtracting and multiplying

You treat a radical just like you did a variable.

You could add x's together (2x+3x = 5x)

And y's together (4y+10y=14y)

So you can add

$$\sqrt{5} + 2\sqrt{5} = 3\sqrt{5}$$

and

$$10\sqrt{2} + 2\sqrt{2} = 12\sqrt{2}$$

Ex 1 page 563
Add and subtract

a) $3\sqrt{5}+4\sqrt{5}=7\sqrt{5}$

b) $\sqrt[4]{w}-6\sqrt[4]{w}=-5\sqrt[4]{w}$

c) $\sqrt{3}+\sqrt{5}-4\sqrt{3}+6\sqrt{5}=-3\sqrt{3}+7\sqrt{5}$

d) $3\sqrt[3]{6x}+2\sqrt[3]{x}+\sqrt[3]{6x}+\sqrt[3]{x}=4\sqrt[3]{6x}+3\sqrt[3]{x}$

** Ex. 5-16**

Ex 2 pg 563
Simplifying then combining

a) $\sqrt{8}+\sqrt{18}=\sqrt{4}\cdot\sqrt{2}+\sqrt{9}\cdot\sqrt{2}=2\sqrt{2}+3\sqrt{2}=5\sqrt{2}$

$note\sqrt{8}+\sqrt{18}\neq\sqrt{26}!!!$

b) $\sqrt{2x^3}-\sqrt{4x^2}+5\sqrt{18x^3}=\sqrt{x^2}\cdot\sqrt{2x}-2x+5\cdot\sqrt{9x^2}\cdot\sqrt{2x}=$

$=x\sqrt{2x}-2x+15x\sqrt{2x}=16x\sqrt{2x}-2x$

c) $\sqrt[3]{16x^4y^3}-\sqrt[3]{54x^4y^3}=\sqrt[3]{16x^4y^3}\cdot\sqrt[3]{2x}-\sqrt[3]{54x^4y^3}\cdot\sqrt[3]{2x}=$

$=2xy\cdot\sqrt[3]{2x}-3xy\cdot\sqrt[3]{2x}=-xy\cdot\sqrt[3]{2x}$

Ex. 17-32

Ex 3 Multiplying radicals with the same index pg 564 **33-46**

a) $5\sqrt{6}\cdot 4\sqrt{3} = 5\cdot 4\cdot \sqrt{6}\cdot \sqrt{3} = 20\sqrt{18} = 20\cdot 3\sqrt{2} = 60\sqrt{2}$

b) $\sqrt{3a^2}\cdot\sqrt{6a} = \sqrt{18a^3} = \sqrt{9a^2}\cdot\sqrt{2a} = 3a\sqrt{2a}$

c) $\sqrt[3]{4}\cdot\sqrt[3]{4} = \sqrt[3]{16} = \sqrt[3]{8}\cdot\sqrt[3]{2} = 2\sqrt[3]{2}$

d) $\sqrt[4]{\frac{x^3}{2}}\cdot\sqrt[4]{\frac{x^2}{8}} = \sqrt[4]{\frac{x^5}{16}} = \frac{\sqrt[4]{x^4}\cdot\sqrt[4]{x}}{\sqrt[4]{16}} = \frac{x\sqrt[4]{x}}{2}$

Ex 4 Multiplying radicals pg 565

** Ex. 47-60**

a) $3\sqrt{2}(4\sqrt{2}-\sqrt{3}) = 3\sqrt{2}\cdot 4\sqrt{2} - 3\sqrt{2}\cdot\sqrt{3} = 12\cdot 2 - 3\sqrt{6} = 24 - 3\sqrt{6}$

b) $\sqrt[3]{a}(\sqrt[3]{a}-\sqrt[3]{a^2}) = \sqrt[3]{a^2} - \sqrt[3]{a^3} = \sqrt[3]{a^2} - a$

c) $\left(2\sqrt{3}+\sqrt{5}\right)\left(3\sqrt{3}-2\sqrt{5}\right) --> FOIL -->$

$= 2\sqrt{3}\cdot 3\sqrt{3} - 2\sqrt{3}\cdot 2\sqrt{5} + \sqrt{5}\cdot 3\sqrt{3} - \sqrt{5}\cdot 2\sqrt{5} =$

$= 18 - 4\sqrt{15} + 3\sqrt{15} - 10 = 8 - \sqrt{15}$

d) $\left(3+\sqrt{x-9}\right)^2 = 3^2 + 2\cdot 3\sqrt{x-9} + (\sqrt{x-9})^2 =$

$= 9 + 6\sqrt{x-9} + x - 9 = x + 6\sqrt{x-9}$

One of those special products - reminder

We’ve looked a lot at:

$(a+b)(a-b) = a^2-b^2$

So if you multiply two things like this with radicals, just square the first and last and subtract them!

Example 6 Multiplying Conjugates pg566 **Ex. 61-70**

a) $(2+3\sqrt{5})(2-3\sqrt{5}) = 2^2 - (3\sqrt{5})^2 = 4-45 = -41$

b) $(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2}) = 3-2 = 1$

c) $(\sqrt{2x}-\sqrt{y})(\sqrt{2x}+\sqrt{y}) = 2x - y$

Example 7 Why not even mix the indices…why the heck not?

Remember $a^m \cdot a^n = a^{m+n}$

a) $\sqrt[3]{2} \cdot \sqrt[4]{2} = 2^{1/3} \cdot 2^{1/4} = 2^{7/12} = \sqrt[12]{2^7} = \sqrt[12]{128}$

b) $\sqrt[3]{2} \cdot \sqrt{3} = 2^{1/3} \cdot 3^{1/2} = 2^{2/6} \cdot 3^{3/6} = \sqrt[6]{2^2} \cdot \sqrt[6]{3^3} = \sqrt[6]{2^2 \cdot 3^3} = \sqrt[6]{108}$

Section 9.3 Adding and stuff

- Definitions Q1-Q4
- Addin' and Subtractin' Q5-32
- Multiplying Q33-Q60
- Conjugates Q61-Q70
- Multiplying different indices Q71-78
- Everything Q79-110
- Word problems Q127-136

9.4 Quotients and Denominator Problems

Remember:

$$\sqrt{2} \cdot \sqrt{2} = 2$$

$$\sqrt[3]{2} \cdot \sqrt[3]{2} \cdot \sqrt[3]{2} = 2$$

Example 1 pg 570 **Ex 1-8**

Fix the denominator – no radicals!

a) $$\frac{\sqrt{3}}{\sqrt{5}} = \frac{\sqrt{3}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{15}}{5}$$

b) $$\frac{3}{\sqrt[3]{2}} = \frac{3}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{4}}{\sqrt[3]{4}} = \frac{3\sqrt[3]{4}}{\sqrt[3]{8}} = \frac{3\sqrt[3]{4}}{2}$$

Step by Step help

A radical expression of index n is in simplified form if it has:

1. No perfect nth power as factors of the radicand
2. No fractions inside the radical, and
3. No radicals in the denominator

Example 2 – Simplifying Radicals

a) $$\frac{\sqrt{10}}{\sqrt{6}} = \frac{\sqrt{10}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{60}}{6} = \frac{\sqrt{4}\sqrt{15}}{6} = \frac{2\sqrt{15}}{6} = \frac{\sqrt{15}}{3}$$

b) $$\sqrt[3]{\frac{5}{9}} = \frac{\sqrt[3]{5}}{\sqrt[3]{9}} = \frac{\sqrt[3]{5}}{\sqrt[3]{9}} \cdot \frac{\sqrt[3]{3}}{\sqrt[3]{3}} = \frac{\sqrt[3]{15}}{\sqrt[3]{27}} = \frac{\sqrt[3]{3}}{3}$$

Ex. 9-18

Example 3- Doing the same with letters. **Ex. 19-28**

a) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \cdot \frac{\sqrt{b}}{\sqrt{b}} = \frac{\sqrt{ab}}{b}$

b) $\sqrt{\frac{x^3}{y^5}} = \frac{\sqrt{x^3}}{\sqrt{y^5}} = \frac{\sqrt{x^2}}{\sqrt{y^4}} \cdot \frac{\sqrt{x}}{\sqrt{y}} = \frac{x\sqrt{x}}{y^2\sqrt{y}} = \frac{x\sqrt{x}}{y^2\sqrt{y}} \cdot \frac{\sqrt{y}}{\sqrt{y}} = \frac{x\sqrt{xy}}{y^2 \cdot y} = \frac{x\sqrt{xy}}{y^3}$

c) $\sqrt[3]{\frac{x}{y}} = \frac{\sqrt[3]{x}}{\sqrt[3]{y}} = \frac{\sqrt[3]{x}}{\sqrt[3]{y}} \cdot \frac{\sqrt[3]{y^2}}{\sqrt[3]{y^2}} = \frac{\sqrt[3]{xy^2}}{\sqrt[3]{y^3}} = \frac{\sqrt[3]{xy^2}}{y}$

Dividing Radicals

$$\sqrt[n]{a} \div \sqrt[n]{b} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

Example 4 Dividing w/same index

a) $\sqrt{10} \div \sqrt{5} = \dfrac{\sqrt{10}}{\sqrt{5}} = \sqrt{\dfrac{10}{5}} = \sqrt{2}$

b) $\left(3\sqrt{2}\right) \div \left(2\sqrt{3}\right) = \dfrac{3\sqrt{2}}{2\sqrt{3}} = \dfrac{3\sqrt{2}}{2\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{3\sqrt{6}}{2 \cdot 3} = \dfrac{\sqrt{6}}{2}$

c) $\sqrt[3]{10x^2} \div \sqrt[3]{5x} = \dfrac{\sqrt[3]{10x^2}}{\sqrt[3]{5x}} = \sqrt[3]{\dfrac{10x^2}{5x}} = \sqrt[3]{2x}$

Ex. 29-36

Ex 5 – Or simplify BEFORE you divide

a) $$\sqrt{12} \div \sqrt{72x} = \frac{\sqrt{4} \cdot \sqrt{3}}{\sqrt{36} \cdot \sqrt{2x}} = \frac{2\sqrt{3}}{6\sqrt{2x}} = \frac{\sqrt{3} \cdot \sqrt{2x}}{3\sqrt{2x} \cdot \sqrt{2x}} = \frac{\sqrt{6x}}{6x}$$

b) $$\sqrt[4]{16a} + \sqrt[4]{a^5} = \frac{\sqrt[4]{16} \cdot \sqrt[4]{a}}{\sqrt[4]{a^4} \cdot \sqrt[4]{a}} = \frac{\sqrt[4]{16}}{\sqrt[4]{a^4}} = \frac{2}{a}$$

** Ex 37-44**

Ex 6 – Simplifying radical expressions

a) $$\frac{4-\sqrt{12}}{4}=\frac{4-2\sqrt{3}}{4}=\frac{2(2-\sqrt{3})}{2\cdot 2}=\frac{2-\sqrt{3}}{2}$$

b) $$\frac{-6+\sqrt{20}}{-2}=\frac{-6+2\sqrt{5}}{-2}=\frac{-2(3-\sqrt{5})}{-2}=3-\sqrt{5}$$

**Ex. 45-48

Ex 7 – Rationalizing the denominator using conjugates

a) $$\frac{2+\sqrt{3}}{4-\sqrt{3}} = \frac{(2+\sqrt{3})}{(4-\sqrt{3})}\cdot\frac{(4-\sqrt{3})}{(4-\sqrt{3})} = \frac{8+6\sqrt{3}+3}{13} = \frac{11+6\sqrt{3}}{13}$$

b) $$\frac{\sqrt{5}}{\sqrt{6}+\sqrt{2}} = \frac{\sqrt{5}}{\sqrt{6}+\sqrt{2}}\cdot\frac{(\sqrt{6}+\sqrt{2})}{(\sqrt{6}+\sqrt{2})} = \frac{\sqrt{30}-\sqrt{10}}{4}$$

** Ex. 49-58**

All the tricks again…now things to powers Ex. 8 pg 575

a) $(5\sqrt{2})^3 = 5^3(\sqrt{2})^3 = 125\sqrt{8} = 125\cdot 2\sqrt{2} = 250\sqrt{2}$

b) $(2\sqrt{x^3})^4 = 2^4\sqrt{(x^3)^4} = 16\sqrt{x^{12}} = 16x^6$

c) $(3w\sqrt[3]{2w})^3 = 3^3 w^3(\sqrt[3]{2w})^3 = 27w^3(2w) = 54w^4$

d) $(2t\sqrt[4]{3t})^3 = 2^3 t^3(\sqrt[4]{3t})^3 = 8t^3\sqrt[4]{27t^3}$

Ex. 59-70

Section 9.4 Dividing Radical Stuff

- Rationalize the Denominator Q1-8
- Simplifying Radicals Q9-28
- Dividing Radicals Q29-Q48
- Using Conjugates Q49-Q58
- Powers of Powers Q59-70
- Everything Q71-Q108
- Word problems Q109-110

Quantum Leap again to Section 9.5 this time

- Now we do the SAME thing but we are solving equations and working with word problems with the RADICALS making a return.

$$\sqrt{}$$

The odd – root Property (aren't they ALL odd?)

- Remember… $(-2)^3 = -8$ and $2^3 = 8$
- So the solution of $x^3 = 8$ is 2

and the solution of $x^3 = -8$ is -2

- Because there is only one real odd root of each real number.

(Flash card time)
The Odd-Root Property

- If *n* is an odd positive integer

$x^n = k$ is equivalent to $x = \sqrt[n]{k}$

for any real number k

Try it on for size
Example 1 page 579

- a) $x^3 = 27, x = \sqrt[3]{27}, x = 3$

- b) $x^5 + 32 = 0, x^5 = -32, x = \sqrt[5]{-32}, x = -2$

- c) $(x-2)^2 = 24, x - 2 = \sqrt[3]{24}, x = 2 + 2\sqrt[3]{3}$

Ex. 5-12

The Even-Root Property

- Oooh spooky.
- If you have $x^2=4$ the answer is 2. Right?
- Bzzzt!
- We know $(2)^2=4$. Great! BUT $(-2)^2=4$ also.
- So the solution is $x=\{2,-2\}$
- Another way to write this is $x=\pm 2$
- So in $x^4=16$, $x=\pm 4$
- And in $x^6=5$ is $x=\pm \sqrt[6]{5}$

Leaving the book for a page

- Remember John's fractional exponent trick?
- The solution of x^2 is $\sqrt{x}$

- Let's solve it:

we have

That's *why*

$$x^2 = 1$$

$$(x^2)^{1/2} = (1)^{1/2}$$

$$x^{\frac{1}{2}\cdot 2} = 1$$

$$x = \pm 1$$

$$x^2 = 1$$

$$\sqrt{(x^2)} = \sqrt{(1)}$$

$$x = \pm 1$$

And the last one from the previous slide was…

- And in $x^6=5$ is $x=\pm\sqrt[6]{5}$

$$x^6 = 5$$
$$(x^6)^{1/6} = (5)^{1/6}$$
$$x^{\frac{1}{6}\cdot 6} = (5)^{\frac{1}{6}}$$
$$x = \pm 5^{\frac{1}{6}}$$

$$x^6 = 5$$
$$\sqrt[6]{x^6} = \sqrt[6]{5}$$
$$x = \pm\sqrt[6]{5}$$

The Even root problem

- In short, if the number inside the even root is positive, you have a + and – answer.
- If it's zero, the answer it just zero.
- If it's negative, you have no solution (in this universe… it is an imaginary number).

Technically the Even Root Property looks like

- If k > 0, then $x^n = k$ is equivalent to $x = \pm\sqrt[n]{k}$
- If k = = then $x^n = 0$ is equivalent to x=0
- If k<0, then $x^n = k$ has no real solution

Example 2 page 580
using the EVEN root property

- a) $x^2 = 10, x = \pm\sqrt{10}$

  $so\_the\_solution\_set\_is = \{-\sqrt{10}, \sqrt{10}\} or \{\pm\sqrt{10}\}$

- b) $w^8=0$ so $w=0$

- c) $x^4= -4$ has no real solution

(to the physicists = 2i, to engineers it is 2j)

Ex. 13-18

Example 3 page 581
Using this same property

a) $(x-3)^2=4$

$x-3=2$ or $x-3=-2$

$x=5$ or $x=1$ The solution set is {1,5}

Example 3b

b)

$$2(x-5)^2-7=0$$

$$2(x-5)^2=7$$

$$(x-5)^2=\frac{7}{2}$$

$$x-5=\sqrt{\frac{7}{2}}, or \_\_ x-5=-\sqrt{\frac{7}{2}}$$

$$multiply \_ \sqrt{\frac{7}{2}} by \sqrt{\frac{2}{2}} \rightarrow \sqrt{\frac{7}{2}}\sqrt{\frac{2}{2}}=\sqrt{\frac{14}{4}}=\frac{\sqrt{14}}{2}$$

$$x=5+\frac{\sqrt{14}}{2}, or \_\_ x=5-\frac{\sqrt{14}}{2}$$

$$x=\frac{10}{2}+\frac{\sqrt{14}}{2}, or \_\_ x=\frac{10}{2}-\frac{\sqrt{14}}{2}$$

$$x=\frac{10+\sqrt{14}}{2}, or \_\_ x=\frac{10-\sqrt{14}}{2}$$

Example 3c

- $x^4-1=80$
- $x^4=81$
- $x=\pm\sqrt[3]{81}=\pm3$

- So the solution set is {-3,3}

** Ex 19-28**

Nonequivalent solutions or Extraneous Solutions

- When you solve an equation by squaring both sides, you can get answers that DON'T satisfy the equation you are working with.
- These are extraneous. Throw them out!

Example 4

- Solve:
- a) $\sqrt{2x-3}-5=0,$

$$\sqrt{2x-3}=5$$

$$(\sqrt{2x-3})^2=5^2$$

$$2x-3=25$$

$$2x=28$$

$$x=14\_or\_\{14\}$$

b) $\sqrt[3]{3x+5}=\sqrt[3]{x-1},$

$$\left(\sqrt[3]{3x+5}\right)^3=\left(\sqrt[3]{x-1}\right)^3$$

$$3x+5=x-1$$

$$2x=-6$$

$$x=-3\_or\_\{-3\}$$

Ex 4c

Ex. 29-48

$$\sqrt{3x+18} = x$$

$$(\sqrt{3x+18})^2 = x^2$$

$$3x+18 = x^2$$

$$-x^2+3x+18=0$$

$$(x-6)(x+3)=0$$

$$x-6=0 \quad or \quad x+3=0$$

$$x=6 \quad or \quad x=-3$$

Checking

$$\sqrt{3(-3)+18} = -3 \qquad \sqrt{3(6)+18} = 6$$

$$\sqrt{9} = -3 \qquad \sqrt{36} = 6$$

$$3 \neq -3 \quad bad \quad answer! \qquad 6 = 6 \quad good!!$$

And sometimes you need to square both sides twice EX5

Ex. 49-84

$$\sqrt{5x-1}-\sqrt{x+2}=1$$
$$\sqrt{5x-1}=1+\sqrt{x+2}$$
$$\left(\sqrt{5x-1}\right)^2=\left(1+\sqrt{x+2}\right)^2$$
$$5x-1=1+2\sqrt{x+2}+x+2$$
$$5x-1=3+x+2\sqrt{x+2}$$
$$4x-4=2\sqrt{x+2}$$
$$2x-2=\sqrt{x+2}$$
$$\left(2x-2\right)^2=\left(\sqrt{x+2}\right)^2$$
$$4x^2-8x+4=x+2$$
$$4x^2-9x+2=0$$
$$(4x-1)(x-2)=0$$
$$4x-1=0 \quad or \quad x-2=0$$
$$x=\frac{1}{4} \quad or \quad x=2$$

Checking

$$x=\frac{1}{4} \quad or \quad x=2$$

$$Plug\_it\_in\_to\_\sqrt{5x-1}-\sqrt{x+2}=1$$

$$\sqrt{5\cdot\frac{1}{4}-1}-\sqrt{\frac{1}{4}+2}=\sqrt{\frac{1}{4}}-\sqrt{\frac{9}{4}}=\frac{1}{2}-\frac{3}{2}=1$$

$$\sqrt{5\cdot 2-1}-\sqrt{2+2}=\sqrt{9}-\sqrt{4}=3-2=1$$

Example 6 page 584 **Ex. 65-76**

- a) $x^{\frac{2}{3}} = 4$

$$\left(x^{\frac{2}{3}}\right)^3 = 4^3$$

$$x^2 = 64$$

$$x = 8 \quad or \quad -8$$

$$Both\ work, so = \{-8, 8\}$$

b) $(w-1)^{-\frac{2}{5}} = 4$

$$\left[(w-1)^{-\frac{2}{5}}\right]^{-5} = [4]^{-5}$$

$$(w-1)^2 = \frac{1}{1024}$$

$$w-1 = \pm\sqrt{\frac{1}{1024}}$$

$$w-1 = \frac{1}{32} \quad or \quad w-1 = -\frac{1}{32}$$

$$w = \frac{33}{32} \quad or \quad w = \frac{31}{32}$$

$$Both\ work, so = \{\frac{31}{32}, \frac{33}{32}\}$$

Ex 7 page 585
Not all good things have a solution…

- $(2t-3)^{-2/3}=-1$
- $[(2t-3)^{-2/3}]^{-3}=(-1)^{-3}$
- $(2t-3)^{2}=-1$
- Error! We can’t take the square root of this!
- There is no solution in this universe.

Ex 77-78

Aid in Solving these things

1. In raising each side of an equation to an even power, we can create equations that give extraneous solutions. Check em!
2. When applying the even-root property, remember that there is a positive and a negative root for any positive real number.
3. For equations with rational exponents, raise each side to a positive or negative integral power first, then apply the even- or odd- root property. (Postive fraction – raise to a positive power; negative fraction – raise to a negative power.)

And… back to a few applications…

- The distance formula
- If you have a triangle with points (x_1,y_2) and (x_2,y_2) you can use the Pythagorean theorem

to get the distance

$a^2+b^2=c^2$ becomes

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 8 page 585

- Looking Figure 9.1 we want to know the distance from first to third base when the bases are 90feet apart.

$$x^2 = 90^2 + 90^2$$

$$x^2 = 8100 + 8100$$

$$x^2 = 16,200$$

$$x = \pm\sqrt{16,200} = \pm 90\sqrt{2}$$

Then we ignore the negative answer.

Putting out fires in section 9.5

- Some definitions Q1-Q4
- Solving things with radials with one answer Q5-Q12
- Solving for two or no answers Q13-28
- Solving and checking for extraneous answers Q29-64
- Solving Q65-98
- Word problems Q99-124

Changing Chapters…

Chapter 10.1-10.3

Putting it all together, factoring to graphing. The big sum up.

Section 10.1 Factoring and Completing the Square

- This is MORE of the same. Nothing new EXCEPT square roots may show up.
- If you keep your head about you, this will go down like castor oil.

Review of Factoring

- $ax^2+bx+c=0$
- Where a,b,c are real and
- a isn't equal to 0 (or that's cheating).

Review of the Cookbook

1. Write the equation with 0 on the right hand side (stuff=0)
2. Factor the left hand side.
3. Use the zero factor property to set each factor equal to zero.
4. Solve the simplest equations.
5. Check the answers in the original equation.

Example 1 page 610
Solving a quadratic equation by factoring

Ex. 5-14

$$3x^2 - 4x - 15 = 0$$

$$(3x+5)(x-3) = 0$$

$$3x+5=0\_\_\_\_or\_\_\_\_x-3=0$$

$$3x=-5\_\_\_\_\_or\_\_\_\_x=3$$

$$x=-\frac{5}{3}$$

$$So\_the\_solution\_set\_is\_\left\{-\frac{5}{3},3\right\}$$

Example 2 pg 611
Review of the Even-Root Property

- This should also go down quickly… since you've done it sooooo much!
- If you solve $(a-1)^2=9$ you get…

$$We\_know\_x^2=k\_is\_also\_x=\pm\sqrt{k}$$

$$(a-1)^2=9$$

$$a-1=\pm\sqrt{9}$$

$$So\_a-1=3\_or\_a-1=-3$$

$$a=4\_or\_a=-2$$

$$\{-2,4\}$$

Ex. 15-24

Completing the Square (making polynomials the way YOU want them)

- Can you make factorable polynomials if you are only given the first two terms?
- What about x^2+6x ?

- To find the last term, remember that you start with two of the things added together that make that middle term that when multiplied together equal that last term.
- Or, in other words, $(b/2)^2$ is your last term.

The rule for finding the last term…

- x^2+bx has a last term that makes the entire polynomial look like:

$$x^2+bx+(b/2)^2$$

Ex 3 pg 612
Raiders of the Lost Term

- a) $$x^2 + 8x + \_\_$$
$$(8/2)^2 = 4 \cdot 4 = 16$$
$$So \quad x^2 + 8x + 16$$

- b) $$x^2 - 5x + \_\_$$
$$(-5/2)^2 = (-5/2)(-5/2) = \frac{25}{4}$$
$$So \quad x^2 - 5x + \frac{25}{4}$$

Ex 3 continued

- c) $$x^2 + \frac{4}{7}x + \_\_$$
  $$(\frac{1}{2} \cdot \frac{4}{7})^2 = (\frac{2}{7})(\frac{2}{7}) = \frac{4}{49}$$
  $$So \quad x^2 + \frac{4}{7}x + \frac{4}{49}$$
- d) $$x^2 - \frac{3}{2}x + \_\_$$
  $$(\frac{1}{2} \cdot \frac{3}{2})^2 = (\frac{3}{4})(\frac{3}{4}) = \frac{9}{16}$$
  $$So \quad x^2 - \frac{3}{2}x + \frac{9}{16}$$

Ex. 25-32

Example 4 pg 612

Remember the perfect square trinomials?

- We're looking for things in the form $a^2+2ab+b^2=(a+b)^2$
- a) $x^2+12x+36 = (x+6)^2$
- b) $y^2-7y+49/4 = (y-7/2)^2$
- c) $z^2-4/3z + 4/9 = (y-2/3)^2$

Ex 33-40

If a=1 then we can complete the squares… Example 5 pg 613

- Given $x^2+6x+5=0$
- The perfect square who's first two terms are x^2+6x is x^2+6x+9
- So we just add 9 to both sides to FORCE this to be a perfect square!
- $x^2+6x+5+9=0+9$
- $x^2+6x+9=9-5$
- $(x+3)^2=4$
- Now we solve it…

Solving Ex 5

$$(x+3)^2 = 4$$

$$x+3 = \pm\sqrt{4} = \pm 2$$

$$x+3 = 2 \quad or \quad x+3 = -2$$

$$x = -1 \quad or \quad x = -5$$

Ex. 41-48

If the coefficient of a isn't 1…

- Too bad. To make this work, you have to MAKE it = 1!!
- So divide both sides in their entirety by whatever is before the a
- For example if you have $2x^2+4x+10=8$
- Then divide EVERYTHING by 2
- Making it $x^2+2x+5=4$ then work on…

The cookbook for these critters: Solving Quadratic Equations by Completing the Squares

1. The coefficient of x^2 must be 1
2. Get only the x^2 and x terms alone on the RHS
3. Add to each side ½ the coefficient of x
4. Factor the left hand side as the square of a binomial
5. Apply the even root property (plus or minus the square root of the remaining number)
6. Solve for x
7. Simplify

Example 6 pg 614
a isn't 1

** Ex.49-50**

- $2x^2+3x-2=0$

$$\frac{2x^2+3x-2}{2}=\frac{0}{2}$$

$$x^2+\frac{3}{2}x-1=0$$

$$x^2+\frac{3}{2}x \quad =1$$

$$x^2+\frac{3}{2}x+\frac{9}{16}=1+\frac{9}{16}$$

$$\left(x+\frac{3}{4}\right)^2=\frac{25}{16}$$

$$x+\frac{3}{4}=\pm\sqrt{\frac{25}{16}}$$

$$x+\frac{3}{4}=\frac{5}{4}\_or\_x+\frac{3}{4}=-\frac{5}{4}$$

$$x=\frac{2}{4}=\frac{1}{2}\_or\_x=-\frac{8}{4}=-2$$

Example 7 pg 615

x^2-3x-6=0

Ex. 51-60

$$x^2-3x-6=0$$

$$x^2-3x \qquad =6$$

$$x^2-3x+\frac{9}{4}=6+\frac{9}{4}$$

$$\left(x-\frac{3}{2}\right)^2=\frac{33}{4}$$

$$x-\frac{3}{2}=\pm\sqrt{\frac{33}{4}}$$

$$x=\frac{3}{2}\pm\sqrt{\frac{33}{4}}$$

$$x=\frac{3\pm\sqrt{33}}{2} \quad or \quad \left\{\frac{3+\sqrt{33}}{2},\frac{3-\sqrt{33}}{2}\right\}$$

Now we'll disguise dishwashing liquid as hand lotion

Ex 8 pg 615 (square first then solve) ** Ex. 61-64**

- Can we deal with square roots in the problem?

$$x+3=\sqrt{153-x}$$
$$(x+3)^2=\left(\sqrt{153-x}\right)^2$$
$$x^2+6x+9=153-x$$
$$x^2+7x-144=0$$
$$(x-9)(x+16)=0$$
$$x-9=0 \quad or \quad x+16=0$$
$$x=9 \quad or \quad x=-16$$

Checking…

$$Root \quad x=9?$$
$$9+3=?\sqrt{153-9}$$
$$12=\sqrt{144} \quad good!$$
$$Root \quad x=-16$$
$$-16+3=?\sqrt{153}$$
$$-13\neq\sqrt{169}=13 \_ BAD!$$
$$Extraneous \quad root$$

Example 9 pg 616

LCD then complete the squares

** Ex. 65-68**

$$\frac{1}{x}+\frac{3}{x-2}=\frac{5}{8}$$

$$The\_LCD\_is=8x(x-2)$$

$$8x(x-2)\frac{1}{x}+8x(x-2)\frac{3}{x-2}=8x(x-2)\frac{5}{8}$$

$$8x-16+24x=5x^2-10x$$

$$32x-16=5x^2-10x$$

$$-5x^2+42x-16=0$$

$$(5x-2)(x-8)=0$$

$$So\quad 5x-2=0\quad or\quad x-8=0$$

$$x=\frac{2}{5}\quad or\quad x=8$$

$$Both\quad work\quad so\quad \left\{\frac{2}{5},8\right\}$$

Toying with the dark side… imaginary solutions!

Example 10 pg 616 **Ex. 69-78**

$$x^2 - 4x + 12 = 0$$

$$x^2 - 4x \quad\_\_\_\_ = -12$$

$$x^2 - 4x + 4 = -12 + 4$$

$$(x-2)^2 = -8$$

$$x - 2 = \pm\sqrt{-8}$$

$$x = 2 \pm i\sqrt{8} = 2 \pm i\sqrt{4 \cdot 2}$$

$$x = 2 \pm 2i\sqrt{2}$$

Section 10.1 Try a completed square on for size!

- Definitions Q1-4
- Review – solve by factoring Q5-14
- Use the even root property Q15-24
- Finish the perfect square trinomial Q25-32
- Factor perfect square trinomials Q33-40
- Solve by completing the square Q41-58
- Potpourri of problems Q59-66
- Complex Answers Q67-90
- Check answers Q91-94
- Word Problems Q95-106

Section 10.2 The Return to the Temple of the Quadratic Formula

- Or… how to get the answer without doing ANYTHING that is hard as what we have already done!!

- The scientific term for it is: Plug'n'Chug

Remember the standard form?

- $ax^2+bx+c=0$

- We can solve for x and always find out what x (or the x's) are.

Developing it…

- We'll just look at it like one would look at the Grand Canyon.
- You can enjoy it and get into it if you're in good shape.

- (where was I going with *this* slide?)

$$ax^2+bx+c=0,$$

$$\frac{ax^2+bx+c}{a}=\frac{0}{a},$$

$$x^2+\frac{b}{a}x+\frac{c}{a}=0,$$

$$x^2+\frac{b}{a}x=-\frac{c}{a},$$

$$the\_square\_of\_1/2\_of\_\frac{b}{a}\_is\_\left(\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}$$

$$x^2+\frac{b}{a}x+\frac{b^2}{4a^2}=-\frac{c}{a}+\frac{b^2}{4a^2},$$

$$\left(x+\frac{b}{2a}\right)^2=+\frac{b^2}{4a^2}-\frac{c}{a}\frac{4a}{4a},$$

$$\left(x+\frac{b}{2a}\right)^2=+\frac{b^2-4ac}{4a^2},$$

$$x+\frac{b}{2a}=\pm\sqrt{\frac{b^2-4ac}{4a^2}},$$

$$x=-\frac{b}{2a}\pm\sqrt{\frac{b^2-4ac}{4a^2}},$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

The Quadratic Formula

- $ax^2+bx+c=0$ where a isn't 0

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 1 pg 623
(become the numbers) **Ex. 7-14**

- $x^2+2x-15=0$
- a=1 b=2   c=-15

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-2\pm\sqrt{2^2-4\cdot1\cdot-15}}{2(1)},$$

$$x=\frac{-2\pm\sqrt{4+60}}{2},$$

$$x=\frac{-2\pm\sqrt{64}}{2},$$

$$x=\frac{-2\pm8}{2},$$

$$So, x=\frac{-2+8}{2}, or, \frac{-2-8}{2},$$

$$x=\{-5,3\}$$

Example 2 pg 623 Only solution

Ex. 15-20

- $4x^2$-12x+9=0
- a=4 b=-12   c=9

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{12 \pm \sqrt{(-12)^2 - 4 \cdot 4 \cdot 9}}{2(4)},$$

$$x = \frac{12 \pm \sqrt{144 - 144}}{2},$$

$$x = \frac{12 \pm 0}{8} = \frac{12}{8} = \frac{3}{2}$$

Example 3 pg. 624
Two irrational solutions **Ex. 21-26**

- $2x^2+16x+3=0$
- a=2 b=16 c=3

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-6\pm\sqrt{(6)^2-4\cdot 2\cdot 3}}{2(2)},$$

$$x=\frac{-6\pm\sqrt{36-24}}{4}=\frac{-6\pm\sqrt{12}}{4},$$

$$x=\frac{-6\pm\sqrt{4\cdot 3}}{4}=\frac{-2\cdot 3\pm 2\sqrt{3}}{2\cdot 2},$$

$$x=\frac{-2\cdot 3\pm 2\sqrt{3}}{2\cdot 2}=\frac{2(-3\pm\sqrt{3})}{2\cdot 2},$$

$$x=\frac{-3\pm\sqrt{3}}{2}$$

Example 4 pg 625

Two imaginary solutions (they are in *elsewhere*)

Ex. 27-32

- $x^2+x+5=0$
- a=1 b=1   c=5

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-1\pm\sqrt{(1)^2-4\cdot1\cdot5}}{2(1)}=\frac{-1\pm\sqrt{-19}}{2},$$

$$x=\frac{-1\pm i\sqrt{19}}{2}, remember\_\sqrt{-1}=i$$

The big picture

- Use the quick reference guide on PAGE 538 for all the different ways to solve

$$ax^2+bx+c=0$$

How many solutions? Look to the *discriminate*.

- From the earlier examples, you get two answers when the b^2-4ac is positive
- You get one answer when b^2-4ac is = 0.
- And no real answers, only imaginary ones, when b^2-4ac is negative.

Example 5 pg. 626 **Ex. 33-48**

a) $x^2-3x-5=0$

$b^2-4ac = (-3)^2-4*1*(-5)=9+20=29$ two real ans.

b) $x^2=3x-9 \rightarrow x^2-3x+9 =0$

$b^2-4ac = (-3)^2-4*1*9= 9-36=-27$ two imag. answers

c) $4x^2-12x+9=0$

$b^2-4ac = (-12)^2-4*4*9= 144-144=0$ One real ans.

Ex 6 - Application pg 626

Ex. 77-96

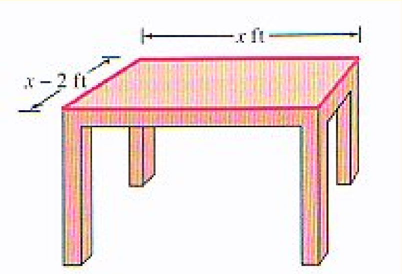


FIGURE 10.1

- If the area of a table is 6 sq ft.

And one side is 2 feet shorter than the other…what are the dimensions?

The Setup x(x-2)=6

Or x^2-2x-6=0

a=1,b=-2,c=-6

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot -6}}{2(1)} = \frac{2 \pm \sqrt{28}}{2},$$

$$x = \frac{2 \pm \sqrt{28}}{2} = \frac{2 \pm \sqrt{4 \cdot 7}}{2} = \frac{2 \pm 2\sqrt{7}}{2},$$

$$x = 1 \pm \sqrt{7}$$

$$Danger\ \ 1 - \sqrt{7}\ \ is\ \ negative - toss\ \ it$$

$$So\ \ x = 1 + \sqrt{7}$$

$$The\ \ sides\ \ are\ \ x\ \ and\ \ x - 2$$

$$Side1: 1 + \sqrt{7}, and \sqrt{7} - 1$$

$$or\ \ 3.65\,feet\ \ and\ \ 1.65\,feet$$

Section 10.2 The Quadratic Formula

- Definitions Q1-Q6
- Solve using the formula Q7-32
- How many solutions? Q33-48
- Solve it the way you want.. Q49-66
- Using a calculator Q67-76
- Word Problems Q77-106

10.3 More-on Quadratic Equations

We just won't get this far this class…sorry!

You can email me and work through it if you want to!

The material below is no longer part of MTH 209

- Go back, there are dragons ahead!

Section 10.4 “I see Quadratic Functions”

Definition, If Y is determined by a formula with X in it, we say *y* is a quadratic function of *x*

$y=ax^2+bx+c$

Example 1 Given a number, what's the other (more plugging in)

- a) $y=x^2-x-6$; given (2,   ), (,0)
- The ()'s are (x,y)
- $y=2^2-2-6=4-2-6=-4$ So the first is (2,-4)
- The other one makes us factor
- $x^2-x-6=0$
- Which is $(x-3)(x+2)=0$ so $x=3$ or -2
- This one gives us two answers (3,0) and (-2,0)

example 1b

- $s=-16t^2+48t+84$ given (0,), (,20)
- This time it's (t,s) inside the ()'s
- The first is $s=-16(0)^2+48(0)+84 = 84$
- It's ordered pair is (0,84)

- The second is $20 =-16t^2+48t+84$ →

$=-16t^2+48t+64$ $=t^2-3t+4$ $=(t-4)(t+1)$ so $t=4$ or $–1$

Giving us (-1,20) and (4,20) as answers.

Graphing. Plug in all values in the universe for x, and see what y is → A parabola.

- They look like this!

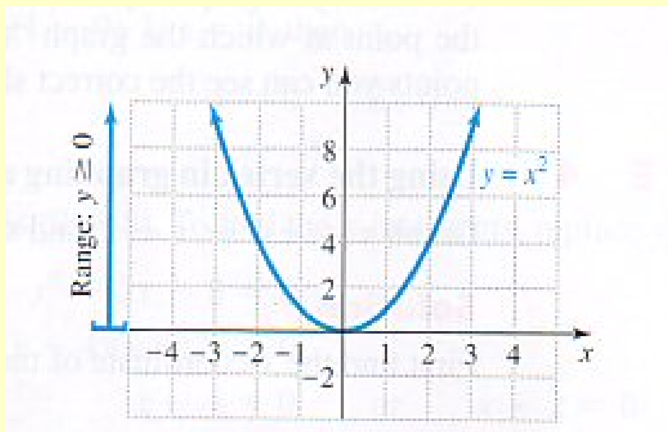


FIGURE 10.2

Example 2 Graphing $y=x^2$

- We can go back to the old – try a few numbers – method.

x	-2	-1	0	1	2
$y=x^2$	4	1	0	1	4

“Can you picture that?”

- It looks like this! With a positive “a” the “U” shape opens upward.

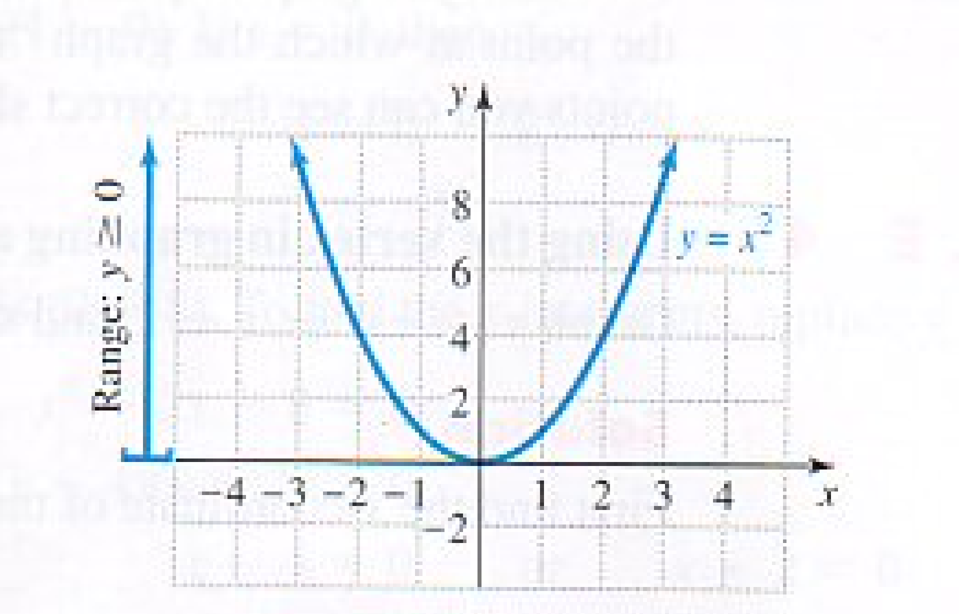


FIGURE 10.2

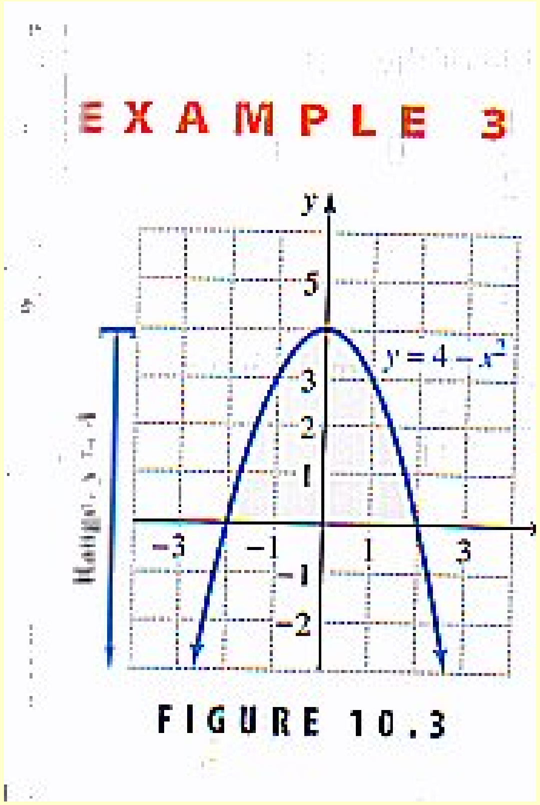
Domain and Range

- The domain is the extent of the graph in X
- The range is the extent of the graph in Y
- In this graph X = $(-\infty, \infty)$ (domain)
- Y is only above and including 0 : $[0, \infty)$

  (range)

Example 3 : $y=4-x^2$

- $y=4-x^2$



x	-2	-1	0	1	2
$y=4-x^2$	0	3	4	3	0

Figuring out more quickly… where is the *VERTEX?*

- The Vertex is the minimum point (if the thing opens upward) or maximum point (if the thing opens downward).
- We can find the vertex by using the front part of:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Mainly $x = \frac{-b}{2a}$

The vertex's above

- For $y=x^2$ for $y=4-x^2$
- The vertex is: (0,0) here it is (0,4)

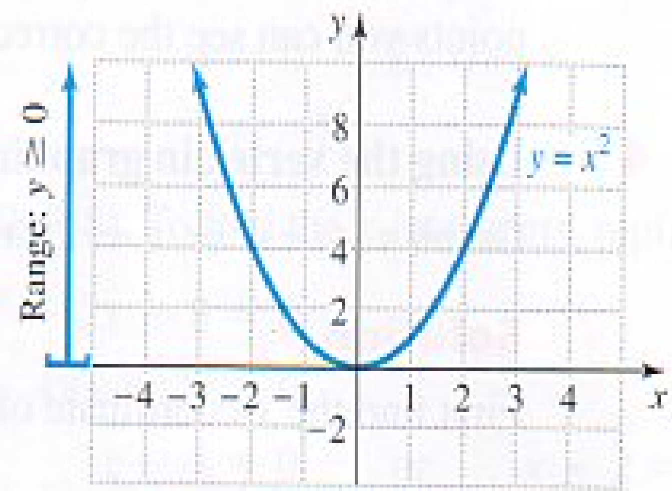


FIGURE 10.2

EXAMPLE 3

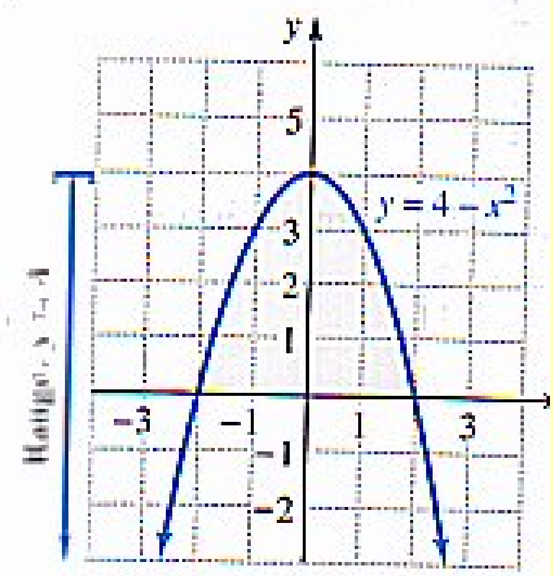


FIGURE 10.3

Example 4 Using $x = \frac{-b}{2a}$

- Graph y=-x^2-x+2

$$x = \frac{-b}{2a} = \frac{-(-1)}{2(-1)} = -\frac{1}{2}$$

$$y\_there\_is...$$

$$y = -x^2 - x + 2$$

$$y = -\left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right) + 2 = \frac{-1}{4} + \frac{1}{4} + 2 = \frac{9}{4}$$

$$The\_Vertex\_is\_(x, y) = \left(-\frac{1}{2}, \frac{9}{4}\right)$$

Example 4 continued

- Plug them numbers in…

x	-2	-1	-1/2	0	1
$y=-x^2-x+2$	0	2	9/4	2	0

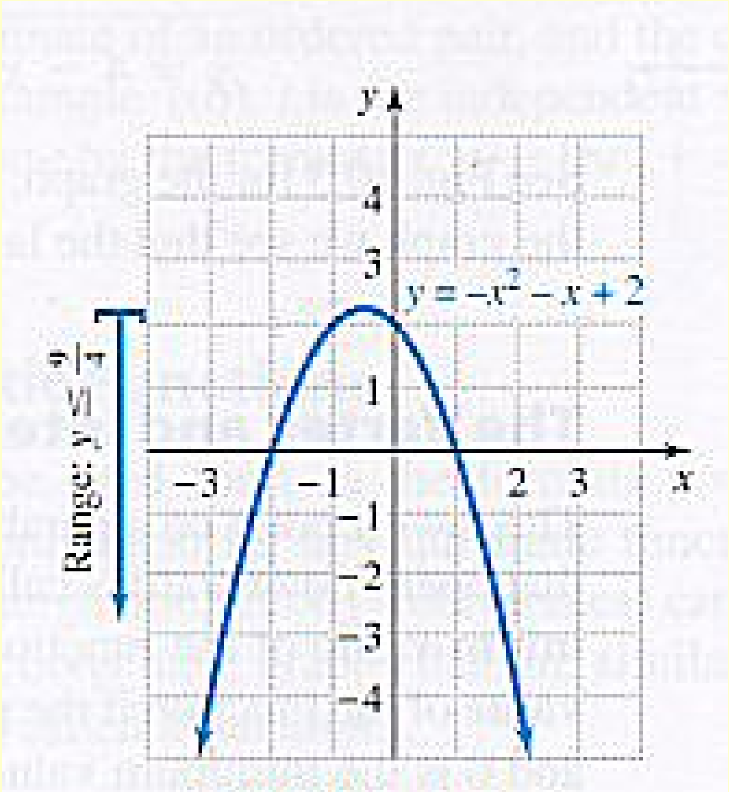


FIGURE 10.4

Example 5 Do it some more…

- a) $y=x^2-2x-8$
- Using $x = \frac{-b}{2a}$ give us x=1, then $y=1^2=2*1-8 = -9$
- The vertex then is (1,-9)
- To find the y-intercept, we can plug in x=0 and find $y=0^2-2*0-8 = -8$ So it's (0,-8)
- To find the x-intercept(s) we can plug in y=0 $x^2-2x-8=0$ or $(x-4)(x+2)=0$ so x=4 or x=-2
- Now we have sleuthed out some points and can plot it…

Example 5a, the graph

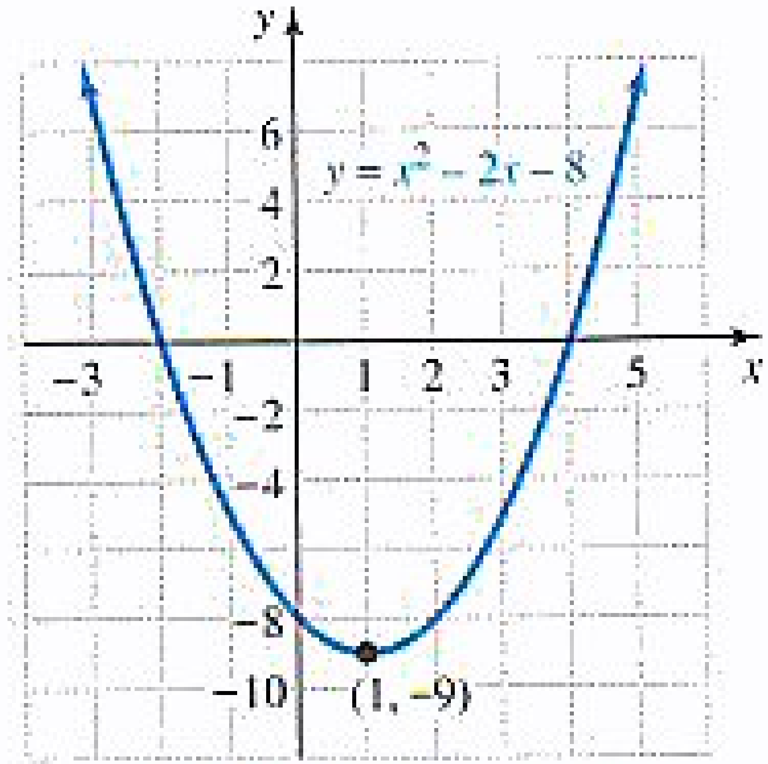


FIGURE 10.5

Example 5b

- a) $s=-16t^2+64t$
- Using $t = \frac{-b}{2a}$ give us t=2, then $s=-16*2^2+64(2) = 64$
- The vertex then is (2,64) (since it is (t,s))
- To find the s-intercept, we can plug in t=0 and find $s=-16(0)^2+64*0 = 0$ So it's (0,0)
- To find the t-intercept(s) we can plug in s=0
  $-16t^2+64t=0$ or $-16t(t-4)=0$ so $-16t=0$ or $t-4=0$

  t=0 or t=4

  the t intercept(s) will be (0,0) and (4,0)
- Now we have sleuthed out some points and can plot it…

Example 5b the picture

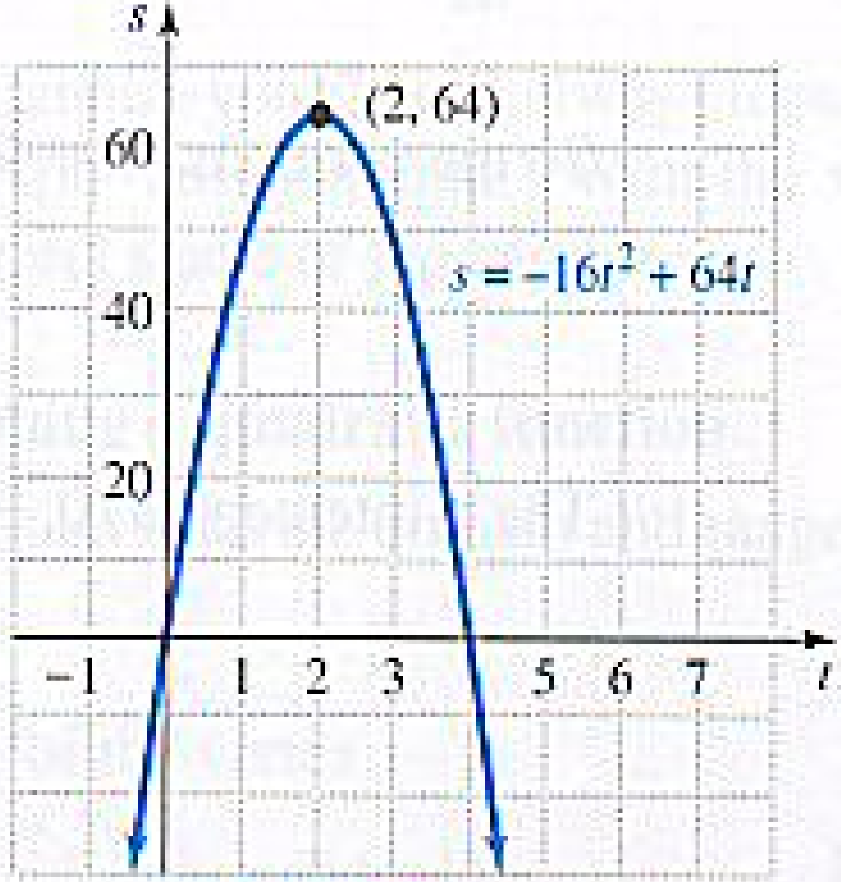


FIGURE 10.6

Graph that quadratic soldiers! Section 10.3

- Definitions Q1-6
- Complete the ordered pairs Q7-10
- Graph the equations Q11-30
- Find the max or min Q31-38
- Word Problems Q39-48 and beyond

New 10.4

Section 10.5 “Alas, Jean Luc. All good things must come to an end.”

- This time we put much of the earlier material together to do your favorite thing!
- We’ll graph quadratic INEQUALITIES on the number line.

- Then you can go run in the beautiful spring air and feel the joyful burden of learning algebra fall off your mathematical shoulders.

Again… it’s just a small step… Quadratic Inequalities

- They look like:

- $ax^2+bx+c > 0$

- where a, b,c are real numbers and a isn’t 0
- We can use $<, \leq, \geq, >$

Example 1

- $x^2+3x-10 > 0$
- $(x+5)(x-2)>0$ The *product* is positive so both may be negative or both may be positive

Value	**Where**	On the number line
x+5=0	if x= -5	Put a 0 above –5
x+5>0	if x>-5	Put + signs to the right of –5
x+5<0	if x<-5	Put – signs to the left of -5

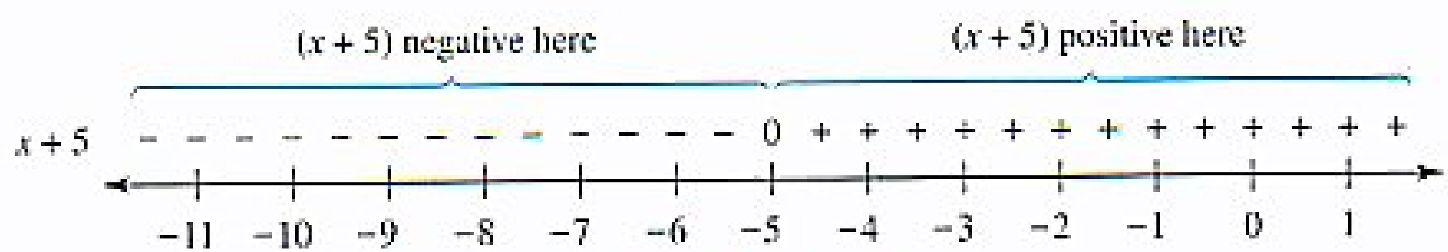


FIGURE 10.9

Example 1 continued

Value	**Where**	On the number line
x-2=0	if x= 2	Put a 0 above 2
x-2>0	if x>2	Put + signs to the right of 2
x-2<0	if x<2	Put – signs to the left of 2

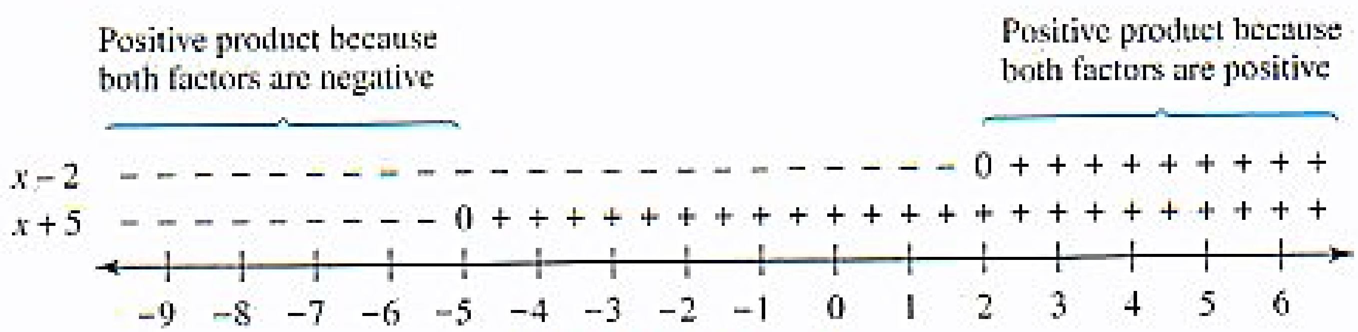


FIGURE 10.10

FIGURE 10.11

Example 2

$$2x^2 + 5x \leq 3$$

$$2x^2 + 5x - 3 \leq 0$$

$$(2x - 1)(x + 3) \leq 0$$

- SOOoooo… one is neg, one is pos. or the opposite.
- 2x-1=0 if x=1/2
- 2x-1>0 if x>1/2
- 2x-1>0 if x<1/2
- x+3=0 if x=-3
- x+3>0 if x>-3
- x+3<0 if x<-3

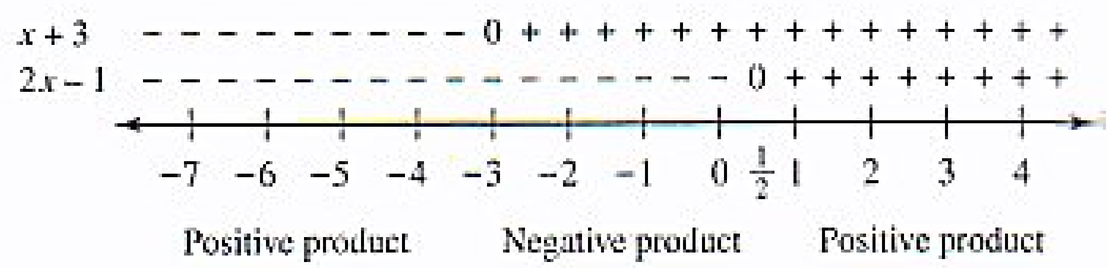


FIGURE 10.12

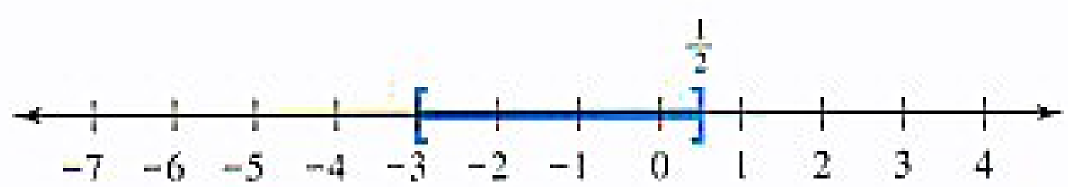


FIGURE 10.13

The cookbook

1. Write the inequality with 0 on the right
2. Factor the quadratic polynomial on the left
3. Make a sign graph showing where each factor is positive, negative or zero.
4. Use the rules for multiplying signed numbers to determine which regions satisfy the original equations.

A reminder about ratios and inequalities

$$\frac{x+2}{x-3} \le 2, \frac{2x-3}{x+5} \le 0, \frac{2}{x+4} \ge \frac{1}{x+1}$$

- You need an LCD to add fractions
- If you multiply by –1 to solve for x, you must reverse the inequality sign (but you don’t have to do anything to the inequality sign if you divide or multiply by a positive number)

Example 3 A rational inequality

$$\frac{x+2}{x-3} \le 2,$$

$$\frac{x+2}{x-3} - \frac{2(x-3)}{x-3} \le 0,$$

$$\frac{x+2}{x-3} - \frac{2x-6}{x-3} \le 0,$$

$$\frac{x+2-2x-6}{x-3} \le 0,$$

$$\frac{-x+8}{x-3} \le 0$$

Ex 3 continued

- $x-3=0$ if $x=3$ $\quad$ $-x+8=0$ if $x=8$
- $x-3>0$ if $x>3$ $\quad$ $-x+8 > 0$ if $x>8$
- $x-3<0$ if $x<3$ $\quad$ $-x+8<0$ if $x<8$

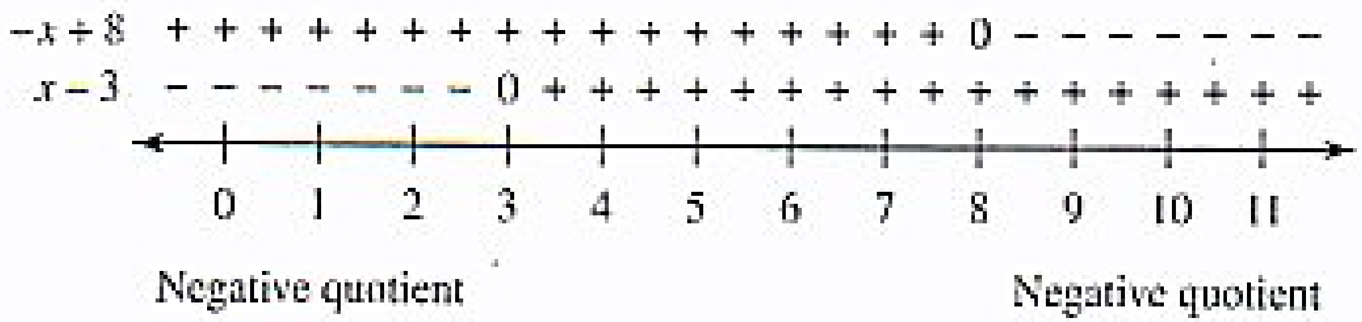


FIGURE 10.14

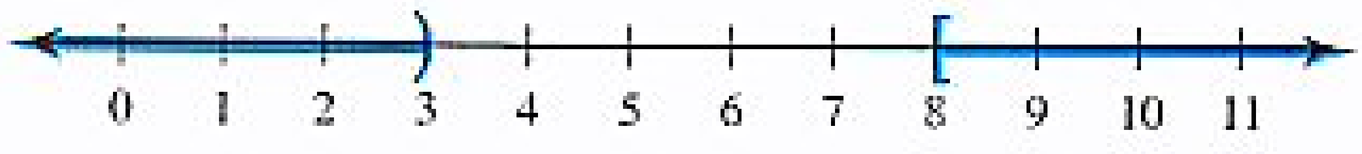


FIGURE 10.15

Example 4 Now put a ratio on both sides…

$$\frac{2}{x+4}-\frac{1}{x+1}\geq 0,$$

$$\frac{2(x+1)}{(x+4)(x+1)}-\frac{1(x+4)}{(x+1)(x+4)}\geq 0,$$

$$\frac{2(x+1)-(x+4)}{(x+4)(x+1)}\geq 0,$$

$$\frac{2x+2-x-4}{(x+4)(x+1)}\geq 0,$$

$$\frac{x-2}{(x+4)(x+1)}\geq 0$$

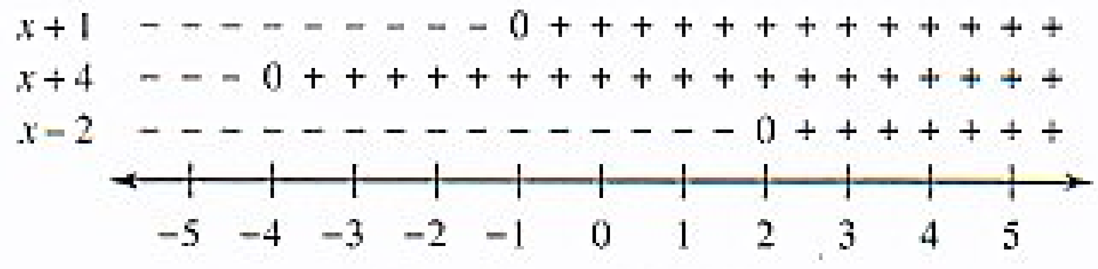


FIGURE 10.16

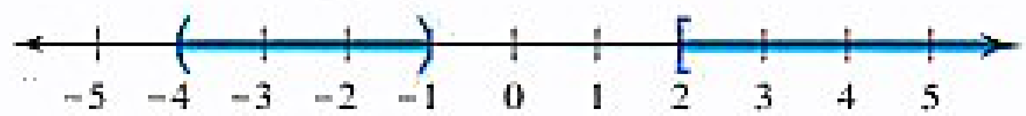


FIGURE 10.17

The cookbook for rational inequalities with a sign graph

1. Rewrite the equation with a 0 on the right hand side
2. Use only addition and subtraction to get an equivalent inequality
3. Factor the numerator and denominator if possible
4. Make a sign graph showing where each factor is positive, negative and zero.
5. Use the rules for multiplying and dividing signed numbers to determine the regions that satisfy the original inequality.

Getting your + and – regions correct. Using a test point.

- Sometimes you can't factor the portions of the quadratic equation.
- Are you stuck?
- Heavens no!
- Why not just use the quadratic equation- then test a few points?

Example 5

- $x^2-4x-6>0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

$$x = \frac{4 \pm \sqrt{16^2 - 4(1)(-6)}}{2(-6)} =$$

$$\frac{4 \pm \sqrt{40}}{2} = \frac{4 \pm 2\sqrt{10}}{2} = 2 \pm \sqrt{10}$$

Example 5 continued

- We can use these points to divide the line by $\left(-\infty, 2-\sqrt{10}\right), \left(2-\sqrt{10}, 2+\sqrt{10}\right), \left(2+\sqrt{10}, \infty\right)$
- Note: $2+\sqrt{10} \approx 5.2, and, 2-\sqrt{10} \approx -1.2$
- So we'll choose –2, 0 and 7 as test points.

We plug those into the first equation and see which are true…

Example 5 finishing it up…

FIGURE 10.18

Test Point	Value of x^2-4x-6 at test point	Sign of x^2-4x-6 in interval of test point
-2	6	Positive
0	-6	Negative
7	15	Positive

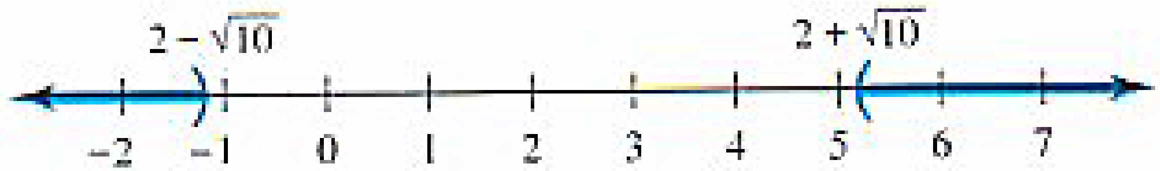


FIGURE 10.19

The quadratic inequalities using Test Points Cookbook

1. Rewrite the inequality with 0 on the right
2. Solve the quadratic equation that results from replacing the inequality symbol with the equals symbol
3. Locate the solutions to the quadratic equation on a number line
4. Select a test point in each interval determined by solving the quadratic equation
5. Test each point in the original quadratic inequality to determine which intervals satisfy the inequality.

Example 6

- After setting up the problem we have the equation… $P=-x^2+80x-1500$
- For what x is her profit positive (x = magazine subscriptions)

$-x^2+80x-1500>0$

$x^2-80x+1500<0$

$(x-30)(x-50)<0$

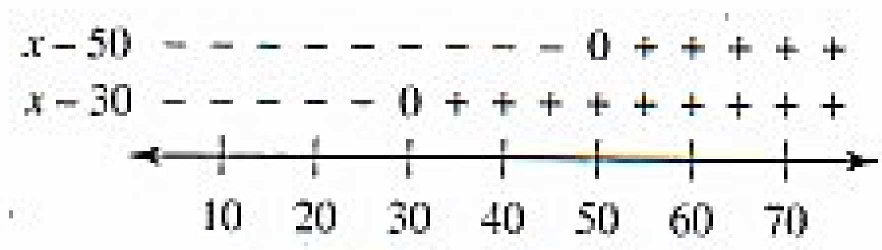


FIGURE 10.20

The
Final Practice!
Section 10.5

- Definitions Q1-4
- Solve each inequality Q5-16
- Do it with rational inequalities Q17-36
- Solve each inequality using interval notation Q37-60
- Word Problems Q61-66

Wrap it up with a final

- Go forth and multiply.
- And factor.
- And find roots.
- etc…

Example 7 Writing in simplified form

- Simplify

a)

$$\frac{\sqrt{10}}{\sqrt{6}} = \frac{\sqrt{10}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{60}}{6} = \frac{\sqrt{4}\sqrt{15}}{6} = \frac{2\sqrt{15}}{6} = \frac{\sqrt{15}}{3}$$

b)

$$\sqrt[3]{\frac{5}{9}} = \frac{\sqrt[3]{5}}{\sqrt[3]{9}} = \frac{\sqrt[3]{5}}{\sqrt[3]{9}} \cdot \frac{\sqrt[3]{3}}{\sqrt[3]{3}} = \frac{\sqrt[3]{15}}{\sqrt[3]{27}} = \frac{\sqrt[3]{15}}{3}$$

Rationalize YOUR denominator

- These square roots (like $\sqrt{2}$ $\sqrt{3}$ and $\sqrt{5}$

are irrational numbers. It is customary to rewrite the fraction with a rational number in the denominator. That is… rationalize it.

- Remember we can always do this:

$$\sqrt{2}\cdot\sqrt{2}=2 \quad \text{since} \quad \sqrt{2}\cdot\sqrt{2}=\sqrt{4}=2$$

Ex 5 Let's rationalize some denominators!

- a) Rationalize: $$\frac{\sqrt{3}}{\sqrt{5}} = \frac{\sqrt{3}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{15}}{5}$$

- b) $$\frac{3}{\sqrt[3]{2}} = \frac{3}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}} = \frac{3\sqrt[3]{4}}{2}$$

Simplified Radical Form for Radicals of Index n

- A radical expression of index *n* is in **simplified form** if it has:
- 1) *no* perfect *n*th powers as factors of the radicand
- 2) *no* fractions inside the radical, and
- 3) *no* radicals in the denominator

Example 7 Of course, we can insert variables into this!

- Simplify… (look for even things you can work with!)
- a)

$$\sqrt{12x^6} = \sqrt{4x^6 \cdot 3} = \sqrt{4x^6} \cdot \sqrt{3} = 2x^3\sqrt{3}$$

- b)

$$\sqrt{98x^5y^9} = \sqrt{49x^4y^8} \cdot \sqrt{2xy} = 7x^2y^4\sqrt{2xy}$$

Example 8 Working with denominators and radicals

- We (traditionally remember) want to get rid of the $\sqrt{}$ in the denominators…
- a) $$\sqrt{\frac{a}{b}} = \sqrt{\frac{a}{b}}\sqrt{\frac{b}{b}} = \frac{\sqrt{ab}}{b}$$
- b)

$$\sqrt{\frac{x^3}{y^5}} = \frac{\sqrt{x^3}}{\sqrt{y^5}} = \frac{\sqrt{x^2}}{\sqrt{y^4}}\frac{\sqrt{x}}{\sqrt{y}} = \frac{x\sqrt{x}}{y^2\sqrt{y}} = \frac{x\sqrt{x}}{y^2\sqrt{y}}\cdot\frac{\sqrt{y}}{\sqrt{y}} = \frac{x\sqrt{xy}}{y^2\cdot y} = \frac{x\sqrt{xy}}{y^3}$$

Of course, why not also complicate things with cube roots and 4th roots

- a) $$\sqrt[3]{40x^8} = \sqrt[3]{8x^6}\,\square\sqrt[3]{5x^2} = 2x^2\sqrt[3]{5x^2}$$

- b) $$\sqrt[4]{x^{12}y^5} = \sqrt[4]{x^{12}y^4}\,\square\sqrt[4]{y} = x^3y\sqrt[4]{y}$$

- c) $$\sqrt[3]{\frac{x}{y}} = \frac{\sqrt[3]{x}}{\sqrt[3]{y}} = \frac{\sqrt[3]{x}}{\sqrt[3]{y}}\frac{\sqrt[3]{y^2}}{\sqrt[3]{y^2}} = \frac{\sqrt[3]{xy^2}}{\sqrt[3]{y^3}} = \frac{\sqrt[3]{xy^2}}{y}$$